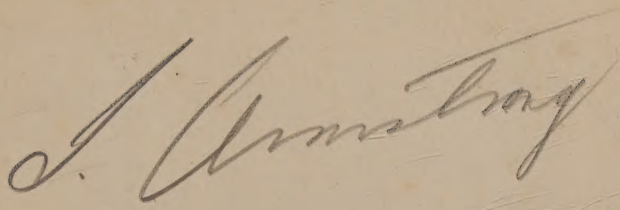
ONTARIO PUBLIC SCHOOL ARITHMETIC

AUTHORIZED BY
THE MINISTER OF EDUCATION
FOR ONTARIO

PRICE 10 CENTS

THE ROBERT SIMPSON COMPANY LIMITED
TORONTO



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THE PUBLIC SCHOOL ARITHMETIC

MEASUREMENT

In each of such terms as: 5 inches, 3 quarts, 7 pounds, $4, three things are named. For example, in 5 inches we have:

(*a*) A **quantity** measured or to be measured, namely, 5 inches.

(*b*) The **unit** used in measuring, namely, one inch.

(*c*) The **number** obtained by counting how many times the unit is found in the quantity, namely, five—the symbol for which is 5.

Number comes from measuring some quantity by a unit.

The unit used is not always the same. It may be one pound, one foot, one dollar, one dozen, one ten, but it is always a part of the quantity measured.

EXERCISE 1

1. Name some quantities you have measured. What unit did you use in each case?

2. What unit would you use to measure: milk? cheese? potatoes? eggs? time?

3. Name some articles which are measured by the peck, the foot, the pair, the head, the twelve, the hundred, the thousand.

4. Measure a quantity of water, using a pint and a quart. What number do you get in each case? How many quarts are there in a gallon?

5. Measure a quantity of sand by the pint and by the gallon. What number do you get in each case? How many pints are there in a gallon, and in a quart?

6. Using the length of your pencil as unit, measure the length of your desk, of the teacher's table, of the black-board.

7. Measure a piece of paper three inches long. Use this as unit and find a quantity of six units. How many two-inch units would there be? How many one-inch units?

8. Name the number and the unit in each of these: 7 cents, 24 couples, 8 twos, 4 ten-dollar bills, 6 groups of five.

9. By measuring, find how many ounces in one pound; how many pounds in 32 ounces. What unit do you use in each case?

10. Measure a peck, using as unit a pint, a gallon, a quart.

11. Measure one yard, using as unit one inch, one foot.

12. Measure these two lines:

(*a*) ———

(*b*) ———————————————

13. Measure the long line, using the short one as unit.

14. If the short line stands for 3 feet, for what will the long one stand?

15. Use objects and measure 12 of them by taking 4 of them as unit; 30 of them by taking 6 as unit.

NOTATION AND NUMERATION

The expression of numbers by symbols is called **notation.**

The reading of numbers which have been expressed in figures is called **numeration.**

EXERCISE 2

1. What name do you give to 10 tens?

2. How many naughts are used in writing one thousand in figures? Where are the naughts written? In what **place** from the right is the one put?

3. Write in figures three thousand; five thousand; nine thousand.

4. Write in figures ten thousand. How many naughts are used in expressing ten thousand? In what place from the right is the 1 put? What separates the 10 from the three ciphers or naughts?

5. Write in figures fifty thousand; seventy thousand; 20 thousand; 80 thousand.

6. How many ciphers must be placed to the right of 15 to make it mean fifteen thousand?

7. Express in figures the following numbers: fourteen thousand, forty-five thousand, nine thousand and two, eight thousand and fifteen, seventy-two thousand and two, twenty-one thousand four hundred and four, four hundred and eight thousand and eighteen, twenty-two thousand seven hundred and ninety-nine, two hundred thousand and four.

8. Read aloud these numbers: 1427, 2341, 4004, 99999, 10010, 334627, 451601, 400400, 303303, 880880, 60062, 205030, 84084, 501010, 304070.

The symbols (1, 2, 3, etc.) which we have been using to express numbers are called **Arabic** numerals, and from the fact that, at first, people counted on their fingers or digits, these symbols are called **digits.**

Each of the first 9 digits has a value of its own, while the cipher, or naught, or zero (0) has no value. It is merely used to keep the other digits in their proper places.

To make easier the reading of a large number written in figures we separate the digits beginning at the right into groups of threes. Each group is called a **period.** Thus, 65437 is written 65,437.

What mark is used to separate the periods?

EXERCISE 3

1. Divide the following into periods: 6274, 92075, 824567, 1000426, 3900457.

2. In 456 what place is occupied by the 6? by the 5? by the 4?

The digit in the *first* place, from the right, is 6 **units** or 6 ones; the digit in the *second* place is 5 **tens**; and the figure in the *third* place is 4 **hundreds.**

In 893,345,617 the first period from the right is called units; the second period, thousands; the third, millions. This is shown in the following table:

Periods	Third	Second	First
Names of groups	Millions	Thousands	Units
Numbers to be read	893	345	617
	495	673	480
	327	705	070
	463	020	005

3. What is the general name for the three digits in the first period? in the second period? in the third period?

4. Divide the number 543212789 into periods, and then read it. Read the second period from the right giving it its name. Read the first. Read the third.

5. Express the following numbers in figures: nine hundred and four thousand six hundred and eighty, eighty million fifty thousand seven hundred and nine, forty thousand and ten, three hundred and three thousand five hundred and twenty-eight, two hundred and three million seven hundred thousand and eighty-seven.

6. How many thousands in 46825? in 289704?

7. Commence at the right and read the number 3640992, digit by digit.

8. How many tens in 870? in 565? How many hundreds in 436? in 815? in 8852? What is left in each case?

9. In 4444444 which of the 4's has the greatest value? How many times as great in value is the second 4 compared

with the first? the third with the second? the fourth with the third? the seventh with the fifth? What is the value of the second 4? of the fifth? Read the second period, the first, the third.

The value of a period or of a figure depends upon the **place** it occupies. Each has, therefore, a **local** value.

10. What is the local value of each *period* in 55555555? of each *digit?*

11. What is the greatest number you can make from the digits 3, 0, 5? from 4, 6, 0? from 8, 2, 9? What is the least?

12. Copy and point off into periods: 3245, 26070, 396281, 900025, 3087650, 11870368.

13. Arrange the following numbers in order, the largest first, then the next, and so on: 39, 405, 550, 4050, 6, 305005, 6203, 471.

14. What is the largest whole number which can be expressed by two figures? What is the smallest?

15. What is the largest number expressed by three figures? the smallest?

16. Write all the three-figure numbers in which one of the two left-hand digits is 7 and the other 3.

17. Write those in which one of the two right-hand figures is 4 and the other 0.

18. Divide into periods, and then write in words, the following: 300001, 50000, 20480, 234888.

19. How many units are there in 12 tens? in 3 tens?

20. How many hundreds are there in 67 tens? in 142 tens? in 1000 tens?

21. How many thousands in 54 hundreds? in 340 tens?

ROMAN NOTATION

What name is given to the symbols 1, 2, 3, 4, 5, 6, 7, 8, 9, 0?

There are other symbols which are sometimes used. These are the capital letters I, V, X, L, C, D, M, and they are called **Roman** numerals.

NOTE: The sign = means *equal* or *equals.*

Their values are: I=1, V=5, X=10, L=50, C=100, D=500, M=1000.

Now, to express the *units'* digit of any number the numerals I, V, and X are used, thus: 1 = I, 2 = II, 3 = III, 4 = IV, 5 = V, 6 = VI, 7 = VII, 8 = VIII, 9 = IX.

To express the *tens'* digit of any number the numerals X, L, and C are used, thus: 10 = X, 20 = XX, 30 = XXX, 40 = XL, 50 = L, 60 = LX, 70 = LXX, 80 = LXXX, 90 = XC.

To express the *hundreds'* digit of any number the numerals C, D, and M are used, thus: 100 = C, 200 = CC, 300 = CCC, 400 = CD, 500 = D, 600 = DC, 700 = DCC, 800 = DCCC, 900 = CM.

To express the *thousands'* digit of any number M is used, thus: 1000 = M, 2000 = MM, 3000 = MMM.

From this it may be seen that when I is placed *before* V or X its value is subtracted from the value of the V or the X. Thus IV = one less than 5, or 4 is one *before* 5. So also 9 is one *before* 10, XL is ten *before* 50, XC is ten *before* 100, CD is 100 *before* 500, and CM is 100 *before* 1000.

Also, when I, or X, or C, or M is repeated its value is repeated. Thus LXXX = 50 and 10 and 10 and 10.

How often may these numerals be repeated?

To express 389 in Roman numerals we have 300 = CCC, 80 = LXXX, and 9 = IX, and thus 389 = CCCLXXXIX. So 2542 = MMDXLII.

EXERCISE 4

1. Read the following and write them in Arabic numerals and in words: XII, XIV, XVIII, XIX, XXIV, XXVI, XXIX, XXXIV, XXXIX, XL, XLIV, LIX, LVI, LXIV, XCVIII, CIX, CXXVII, CXLVI, DC, CD, MCM, MCMX, MMCDLIX, MMMCMXLIV.

2. Write in Roman numerals: 18, 24, 36, 48, 49, 54, 58, 66, 69, 74, 79, 86, 89, 94, 96, 99, 119, 133, 134, 139, 187, 236, 475, 523, 684, 739, 815, 936, 998, 1005, 1487, 1896, 2054, 3865.

3. Write in Arabic numerals: XCI, MCDXLIV, LXIX, CCXIX, XXXVIII, MMMI, XVI, DCCCXCIX, CDIV, MMCMXCIX, XXIV, CXVI, DCXLI, CMX.

4. Write in Roman numerals the number of the year in which the following events took place: Queen Victoria began to reign; George V began to reign; you were born.

5. Have you ever seen 4 written IIII? where?

ADDITION

The sign +, called **plus,** is used to indicate that the numbers between which it is placed are to be added. Thus $27 + 5 = 32$ means "27 and 5 equals 32," and $34 = 29 + 5$ means "34 equals 29 and 5."

What kind of numbers can be added? 5 dollars and 8 dollars can be added and the sum will be 13 dollars. 8 days and 9 peas cannot be added, because the sum would be neither days nor peas.

Only **like** numbers, that is, numbers having the same name, can be added.

The numbers to be added are called **addends** and the number got from their addition is called their **sum.** The sum must have the same name or **denomination** as the addends.

ORAL EXERCISE

Name the sums of the following numbers from left to right, and practise naming them until you can name each the moment your eye rests upon the numbers to be added:

1+1=	2+1=	2+2=	3+1=	3+2=
4+1=	3+3=	4+2=	5+1=	4+3=
5+2=	6+1=	4+4=	5+3=	6+2=
7+1=	5+4=	6+3=	7+2=	8+1=
5+5=	6+4=	7+3=	8+2=	9+1=
6+5=	7+4=	8+3=	9+2=	6+6=
7+5=	8+4=	9+3=	7+6=	8+5=
9+4=	7+7=	8+6=	9+5=	8+7=
9+6=	8+8=	9+7=	9+8=	9+9=

These are the 45 primary "facts" of addition. The "facts" underlined should receive special attention.

There are other "facts" which may be learned from these: for instance, that 8 and 7 make 15; also that 7 taken away from 15 leaves 8; and that 15 less 8 is 7.

When 9 is added to another number in what easy way may the sum be remembered?

All these facts should be thoroughly mastered.

ORAL EXERCISE

Practise the following additions until you can name the results as rapidly as you can count 1, 2, 3, 4, 5, etc.

Add by *twos*, by *threes*, by *fours*, by *fives*, by *sixes*, by *sevens*, by *eights*, by *nines*, and by *tens*, beginning first at 0, then at 1, then at 2, and so on up to 10, until the sum in each case is greater than 100.

In written problems in addition, *units* are usually written under *units*, *tens* under *tens*, *hundreds* under *hundreds*, and so on. This is done in the following:

(1)	(2)	(3)
3754	20234	275 acres
2862	683	146 "
1457	4965	27 "
	68	

In Example 1, what digits represent tens? hundreds?

In Example 2, what figures represent thousands? units? ten thousands?

EXERCISE 5

1. Add 3465, 3279, 6534, 5131, 4268.
2. Add 5732, 6721, 3466, 4269, 6535.
3. Add 2768, 5329, 4605, 475, 16.
4. Add 4671, 272, 45, 7, 625.
5. Add 375, 506, 258, 327, 580, 647, 846.
6. Add 436, 47, 449, 498, 736, 274, 888.
7. Add 625, 494, 742, 673, 574, 654, 638.
8. Add 564, 683, 684, 502, 376, 726, 877.
9. Find the sum of the eight sums of the eight Questions.

NOTE: Addition is usually *checked* or proved by adding the columns from the top down.

The importance of accuracy should never be forgotten. One figure wrong in one out of ten problems is a failure. In actual business life, accuracy is insisted on. Inaccuracy frequently comes from imperfectly formed figures, and the lack of neatness and of proper arrangement.

EXERCISE 6

1. A man bought a sleigh for \$142, a carriage for \$1290, and a pair of horses for \$476. What was the cost of all?

2. A lady paid \$192 for a piano, \$342 for furniture, \$187 for linen, and \$46 for silver. What did she pay for all?

3. A man owns four houses; the first is worth \$47,050, the second \$9,106, the third \$1,492, the fourth \$512. What is the value of them all?

4. What length of fencing will be needed to inclose a lot the four sides of which are 129 feet, 319 feet, 125 feet, and 197 feet?

5. A farmer sold a farm for $6579 and by so doing lost $1724. What did his farm cost him at first?

6. The first three cars of a freight train contained 32,427 pounds each, the next two cars 30,649 pounds each, and the last four cars, 26,427 pounds each. How many pounds of freight were there in the nine cars?

7. The first of five numbers is 6,239, the second is greater than the first by 373, the third is equal to the sum of the first and second, and the fourth is equal to the sum of the second and third. Find the sum of the four numbers.

8. John Brown, James Jones, and Robert Robinson together went into business. Mr. Brown invested $7,937, Mr. Jones $3,987 more than Mr. Brown, and Mr. Robinson $687 more than both. What was the total capital put into the business?

9. On the first day of July, 1907, there were in the County of Bruce 26,409 horses; in Grey, 31,751; in Haliburton, 1,929; in the District of Rainy River, 991; in Thunder Bay District, 568; in York, 25,417; in Wellington, 24,469; in Simcoe, 33,894; in Halton, 9,113; and in Middlesex, 35,075. How many horses were there in all these counties and districts?

10. The value of horses in these counties and districts was as follows: Bruce, $3,221,898; Grey, $3,841,871; Haliburton, $204,474; Rainy River, $128,830; Thunder Bay, $79,520; York, $3,380,461; Wellington, $2,911,811; Simcoe, $4,338,432; Halton, $1,120,899; Middlesex, $4,068,700. What was their total value?

11. In 1906, the value of cheese made in the County of Dundas was $1,189,811; in Dufferin, the value was $44,932; in Essex, $8,200; in Frontenac, $1,123,030; in Peel, $15,022; in Hastings, $1,153,494; in Leeds, $1,164,421; in Muskoka and Nipissing, $4,365; in Welland, $26,203; in Oxford, $1,412,574; and in Lincoln, $68,669. What was the total value of the cheese?

CROP RETURNS OF ONTARIO, 1903-1907

Year	WHEAT		OATS		BARLEY	
	Acres	Bushels	Acres	Bushels	Acres	Bushels
1903...	913,546	21,893,470	2,638,665	109,874,053	709,839	24,378,817
1904...	830,485	12,631,726	2,654,936	102,173,443	772,434	24,567,825
1905 ..	986,329	21,516,588	2,668,416	105,536,572	772,633	24,265,394
1906...	959,032	22,108,774	2,716,711	108,341,455	756,163	25,253,011
1907...	820,678	18,019,142	2,932,509	83,524,301	766,891	21,718,332

12. Examine the table above the line and then find answers to the following questions:

(*a*) How many acres were under wheat during the five years?

(*b*) What was the total amount of wheat (in bushels) grown during the same time?

(*c*) How many acres of oats were there?

(*d*) How many bushels of oats were there?

(*e*) What was the acreage (the whole number of acres) under barley?

(*f*) What was the total yield (in bushels) of barley?

13. The number of children who attended the Public Schools in Ontario during 1907 was as follows: In rural schools, two hundred and thirty-two thousand three hundred and thirty-two; in cities, seventy-three thousand four hundred and ninety-five; in towns, sixty-six thousand three hundred and fifteen: in villages, twenty-six thousand and forty-two. How many children attended the Public Schools of the whole Province?

EXERCISE 7

The following are intended for rapid drill work. To save the time used in copying you may have a slip of paper below the line, at the bottom of each set of addends, and write on that paper the sums as you find them.

Add the following, proving all the results:

(1)	(2)	(3)
124483	69869	316499
438432	22401	202991
179136	61830	412279
158912	50675	773703
173188	76953	3524102
98251	18263	2452542
27341	62483	73192
41273	21276	147628

(4)	(5)	(6)
241400	15755	221295
162393	22328	226821
332823	44151	498496
620923	85564	507254
3188448	335653	493263
1952034	267405	257770
412375	134207	201619
208607	29632	578298

(7)	(8)	(9)
424853	40936	12665
434017	419216	114760
573200	70206	24079
893786	87216	27338
654321	65563	17380
123456	52647	15037
279560	71038	28446
932634	76987	36461
567890	64883	12270
789460	65109	20438
327490	111778	29918

NOTE: If columns are long, business men often write on a separate slip of paper, the partial sums, as here shown, so as to more easily check mistakes.

2357
3467
9987
1707
28
19
23
15
17518

In this example. what is the 15, that is, what is its name? What is the 23? the 19? the 28?

EXERCISE 8 (REVIEW)

1. Find the sum of seventeen, five hundred and sixty-seven thousand and eight, four thousand and eighteen, six hundred and forty-two, five million six hundred and seven thousand and fifty-six, thirty-six million and forty-nine, nine hundred thousand and ten.

2. Draw a line nine inches long, and another nine and a half inches long.

3. The area, or surface, of England is 50,535 square miles; of Scotland, 29,167 square miles; and of Wales, 8,125 square miles. How many square miles are in the three?

4. (*a*) Find the length of your school-room. What is its width?

(*b*) By addition, find how many pecks are in 4 bushels; the number of days in 5 years; the number of minutes in 6 hours.

5. In the year 1910, the area of the different provinces and districts of Canada was as follows: British Columbia, 383,300 square miles; Manitoba, 73,956 square miles; New Brunswick, 28,200 square miles; Nova Scotia, 20,600 square miles; Prince Edward Island, 2,000 square miles; Quebec, 347,350 square miles; Ontario, 222,000 square miles; all the other provinces and districts, leaving Franklin out, two million five hundred and twenty-nine thousand one hundred and forty square miles. What was the area of the whole of Canada?

6. How many inches in 5 feet? How many pints in 4 quarts?

7. A man bought a horse for $275 and a carriage for $342. He sold the horse at a gain of $123 and the carriage at a gain of $65. For how much did he sell both horse and carriage?

8. I bought 3 town lots. The first cost me $325, the second $15 more than the first, and the third as much as both the others. What was the cost of all the lots?

CANADIAN MONEY

The sign $ stands for the word **dollars.** The letter c. stands for **cents.** Thus 17c. is read 17 cents.

When dollars and cents are written together, the cents are separated from the dollars by a full stop (·).

Thus $42 and 58c. is written $42·58.

When the number of cents is less than 10, a cipher must occupy the first place at the right of the full stop which separates the dollars from the cents. Thus $8 and 4c. is written $8·04. Why?

In arranging such numbers for adding, dollars must be placed under dollars, and cents under cents, so that the separating periods or full stops stand under each other, thus:

(1)	(2)	(3)
$376·84	$3497·03	$ 53·70
43·09	69·50	786·10
706·40	240·84	9·08

EXERCISE 9

Read, arrange, and add the numbers in each of the following sets:

1. $4·75, $3083·09, $72·50, $9·32, $384.
2. $93·48, $406·30, $8·07, $5709·80.
3. $500, $93·05, $364·05, $47·09.

4. Express, in figures, the following and then find their sum: nine hundred and six dollars and seventy-five cents, three hundred and twelve dollars and nine cents, eighty-four cents, seven cents, three cents.

5. A furniture dealer sold a bed-room set for $125·86; a book-case for $85·09; and 3 rocking-chairs for $5·75 each. How much did he get for all?

6. A lady, after paying $23·85 for a shawl, $25·50 for a dress, $2·40 for gloves, and $4·08 for other articles, finds that she has $14·28 left. How much had she at first?

7. I bought a house for four thousand two hundred and sixty-eight dollars and ninety cents; furniture for seven hundred and ninety dollars and seven cents; carpeting for three hundred and eighty dollars and sixty cents; and made repairs on the house which cost three hundred and seven dollars and four cents. How much did the whole cost?

EXERCISE 10

Solve these problems, first using pencil; then, if possible, solve them orally:

1. Write, in figures, the number of days in each month of 1909; then find the whole number of days in the year.

2. What month has a different number of days from any of the others?

3. How many months have each 30 days? How many days in all the thirty-day months?

4. How many months have each 31 days? How many days in all these months?

5. How many ounces are there in six pounds? in five pounds?

6. How many ounces in 4 pounds and 8 ounces? in 8 pounds and 3 ounces?

7. How many inches in 10 feet? in 8 feet? in 7 feet and 11 inches? in 9 feet and 9 inches?

8. How many cents in 9 dollars and 89 cents? in 7 dollars and 3 cents?

9. How many months in 8 years?

10. How many school-days in 7 weeks? in 9 weeks?

11. How many work-days are there in 6 weeks?

SUBTRACTION

ORAL EXERCISE

1. If an article is bought for 11 cents, and 25 cents be given to the store-keeper, how much change should he pay back? How does the store-keeper count it?

2. How much will you have to add to 9 to make 16? to 25 to make 34? to 20 to make 80? to 30 to make 75? to 16 to make 30? to 3000 to make 8000?

The number which, added to one number, makes another is called the **difference** between them. That is, the difference between 38 and 20 is 18, because 18 must be added to 20 to make 38.

3. What is the difference between 25 and 17? between 90 and 60? between 35 and 18? between 9000 and 5000? between 1500 and 700? 15000 and 9000? 45 and 38?

FIRST METHOD

(*a*) To find the difference between 879 and 625:

???	879	879
625	625	625
879	???	254

In the first arrangement what digit must be above the 5 so that the 9 in the units' column will be got by adding? What above the 2 to give 7 by adding? What above 6 to give 8? The second arrangement with the smaller number below the larger is the usual way of writing the numbers; and the third shows the same thing with the difference below the line.

(*b*) To find the difference between 623 and 487:

???	623	623
487	487	487
623	???	136

In the first arrangement what digit must be above the 7 so that 3 would be the digit in the units' column after adding? What above the 8 to get 2 in the tens' column when the addition is continued? What above 4 to get 6 in the hundreds' column?

Proceed thus: 7 and **6** make 13; 1 and 8 make 9 and **3** make 12; 1 and 4 make 5 and **1** make 6.

As before, the second arrangement shows the common way of writing down the numbers, and the third shows the difference written *under the line.*

Find the difference between 765 and 324; 9876 and 4532; 485 and 375; 928 and 656; 835 and 687; 7203 and 4725; 26053 and 17264; 80004 and 96432.

NOTE: Work also the Questions in Exercises 11 and 12 by this method.

Finding the difference between two numbers or finding what remains when a smaller number is taken from a larger one, is called **subtraction;** the larger of the two numbers is called the **minuend,** the smaller one is called the **subtrahend,** the number that remains or the difference, is sometimes called the **remainder.**

As + is the sign for addition, so a dash — is the sign for subtraction. This sign — is called the **minus** sign. Minus really means *less,* so that 25—17 means 25 less 17 or 25 lessened by 17.

ORAL EXERCISE

1. Looking at the book, name the remainders:

25—9	33—8	45—6	53—9
76—8	37—8	48—9	32—9
24—9	34—9	44—9	54—9
68—9	56—7	43—5	27—9
31—3	41—3	51—3	61—3
62—7	42—9	40—4	63—8
42—3	54—7	71—8	65—7
60—3	65—9	64—6	87—9
82—8	76—8	28—9	65—6
71—4	95—6	85—7	71—4

2. Beginning first at 100, then at 101, then at 102, then at 103, count backward by *twos,* by *threes,* by *fours,* and so on up to the *nines.*

NOTE: This should be practised until it can be done without hesitation.

SECOND METHOD

From 8765 take 6314. Which of these numbers is the *subtrahend?* Which is the *minuend?* Which should be placed under the other? At which side should you begin to subtract?

$$\begin{array}{r} 8765 \\ 6314 \\ \hline 2451 \end{array}$$

Explanation: Four units from 5 units leaves 1 unit. Then 1 ten from 6 tens leaves 5 tens. Then 3 hundred from 7 hundred leaves 4 hundred. Next 6 thousand from 8 thousand leaves 2 thousand. The *remainder*, therefore, is 2 thousands, 4 hundreds, 5 tens, and 1 unit, or 2,451.

How may the correctness of any work in subtraction be proved?

EXERCISE 11

1. Subtract 53 from 89; 45 from 786; 52 from 674; 632 from 874; 261 from 792; 627 from 798; 532 from 764; 5,431 from 6,982.

2. Find the difference between 7,629 and 4,518; 6,821 and 7,824; 8,542 and 6,131; 8,972 and 6,281.

3. The subtrahend is 613,208, the minuend 725,419. What is the remainder?

4. In each of the following find the remainder:

Minuend 965,420, subtrahend 342,100.
Subtrahend 723,150, minuend 854,267.
Subtrahend 438,320, minuend 549,830.

In taking 3798 from 8356 this difficulty is met: 8 units cannot be taken from 6 units, 9 tens from 5 tens, nor 7 hundreds from 3 hundreds. What is to be done?

(*a*) In the number 8356, if 1 ten be taken from the 5 tens and added to the 6 units, making 16 units, will the value of the whole number be changed? If 1 hundred be taken from the 3 hundred and added to the 4 tens that remain, how many tens shall we have? Now if one of

the 8 thousands be added to the 2 hundreds which remain, how many hundreds shall we have? What we have been doing is taking 1 from some *digit* of the minuend and adding its equivalent, or its equal, to some other *digit* of the minuend, and this will not change the value of the number. The number was, at first, 8356 = 8 thousands + 3 hundreds + 5 tens + 6 units. As changed, it has become 7 thousands + 12 hundreds + 14 tens + 16 units. There will now be no difficulty in subtracting 3798 or 3 thousands + 7 hundreds + 9 tens + 8 units from 8356.

(*b*) Subtract 2347 from 10000.

10000
2347

7 cannot be taken from 0. Take 1000 from 10000 and there will remain 9000. Next, add the 1000 we have taken to the hundreds in the number and we have 10 hundred. Next, take 1 hundred from this 10 hundred and 9 hundred will remain. Adding the 1 hundred to the tens in the number we have 10 tens. Then, taking 1 ten from these 10 tens and adding it to the units we have 9 tens and 10 units.

The minuend was, at first, 10000. It has now become 9000 + 900 + 90 + 10. From this 2347 or 2000 + 300 + 40 + 7 is to be taken, which will leave a remainder of 7000 + 600 + 50 + 3 or 7653.

Usually this is arranged thus:

```
 10000
  2347
 ----
  7653
```

EXERCISE 12

Subtract, explain in full, and prove:

(1)	120 89	(2)	353 168	(3)	8306 7029	(4)	9000 8023
(5)	3245 1679	(6)	2001 1009	(7)	7000 6006	(8)	6111 4789

Subtract and prove:

(9)	(10)	(11)	(12)
35642	87544	90070	8164
12456	64358	13256	3275

ORAL WORK

(13)	(14)	(15)
90543001	6398580044	75000807063
76392132	1296548980	52983878156

NOTE: In Questions 13, 14, and 15, name each figure in the remainder. Thus in Question 13 you should say 9, 6, 8, 0, 5, 1, 4, 1.

Other exercises similar to the foregoing should be done.

EXERCISE 13

Check all your work and give the name or denomination of each answer.

1. What must be added to \$127·65 to make \$143·92? to \$735·75 to make \$926·50? to \$246·50 to make \$432·75? to \$696·75 to make \$823·45?

2. A gentleman received \$65,874. He then paid \$28,598 for building and furnishing a house. How much had he left?

3. A lady bought goods amounting to two dollars and thirty-four cents. She gave a five-dollar bill in payment. What change should she receive?

4. The polar diameter of the earth is 41,707,620 feet, and the equatorial diameter is 41,847,426 feet. Find the difference.

5. How many more inches are there in 16 feet than there are in 9 feet?

6. A house was sold for \$2,387, or for \$98 more than a farm. What was the selling price of the farm?

7. The sum of two numbers is 162,043. One of them is 98,765. What is the other?

8. How many more quarts are there in 27 gallons than there are in 19 gallons?

NOTE: Prove, by subtraction, the accuracy of the answers you get for Questions 9 to 13, and 16, 18 and 21.

9. South America has an area, or surface, of 8,887,794 square miles. North America has an area of 9,349,741 square miles. Which is the larger? How much is it greater in area than the other? What is meant by area?

10. At Waterloo, the Duke of Wellington had an army of 75,686 soldiers. Of these 26,661 were British infantry, 6,877 British artillery, and 33,413 foreign allies. The remainder were British cavalry. How many cavalry were there?

11. A is worth $6,215, B is worth $876 less than A, and C is worth $2,343 more than A and B together. How much are B and C each worth? How much are all three worth?

12. A man sold cattle at a loss of $3,145 and some horses at a gain of $2,587. Did he gain or lose on both transactions? How much?

13. An Italian came to Canada on June 20th, 1889, when he was 23 years old. How old was he on June 20th, 1909?

14. Show, by subtracting, that a farm containing 102 acres can be divided into 6 fields each containing 17 acres.

15. Find, by subtraction, the number of horses at $150 each which can be bought for $750.

16. What number must be added to the sum of sixty-four thousand and one, seven million one thousand and sixty, one thousand seven hundred and nine, seventy-seven thousand and seven, one thousand one hundred, fifty-six thousand seven hundred and eighty-nine, to make the result ten millions?

17. Tom and Harry played at marbles, each having 50. In the first game Harry won all Tom's marbles but 9. He then sold Tom 25. In the second game Tom won 16. How many more marbles had Harry than Tom when they ceased playing?

18. How much is the difference between 628716 and 79019 greater than the sum of 56095, 2800, 10009, 7097, 159, 3000, and 90829?

19. John Stokes bought 3 houses. For the first he gave $5,260, for the second $3,585, and for the third as much as for the first two less $29. He afterward sold the houses for $20,000. What was his gain?

20. A man living in Brockville had in his garden 19 fruit trees, consisting of apple, pear, and cherry trees. The number of cherry and apple trees was 14, and the number of apple and pear trees was 13. How many more pear and cherry trees had he than apple trees?

21. In the cheese factories of Ontario there were 1,692,591,862 pounds of milk used in 1906. In 1897 there were 1,455,937,148 pounds used. What was the increase in the nine years?

22. Write, in words, the two large numbers in Question 21.

23. Give the name of each underlined part of the following number: 437605716. What is the name of the 60? Read the number made up of the 4 and the 1.

24. The Duke of Wellington died in the year 1852, aged 83. Napoleon was born in the same year as the Duke and died in 1821. At what age did Napoleon die?

25. Perform the operations indicated in:

(*a*) $547 - 69 - 89$.

(*b*) $8910 - 702 + 264 - 361 - 1395$.

(*c*) $74891 - (4683 + 4267)$.

(*d*) $77763 + 348 - (9645 - 376)$.

26. If you have no change except a quarter, a half-dollar piece, and a $10 bill, how can you pay an account of $6·55 if the man you owe has only two 10-cent pieces and a number of 2-dollar bills?

27. The distance by rail from Toronto
to London is 115 miles west;
to Sarnia is 170 miles west;
to Belleville is 113 miles east;
to Brockville is 208 miles east:
to Montreal is 333 miles east.
Find the distance of London from each place mentioned after it.

28. David Brown and John Anderson had each $24,950. Brown gave Anderson $8,060, but afterward Anderson gave Brown $16,845. Brown lost so much in speculating that Anderson had $6,200 more than Brown. How much did Brown lose?

29. The following are a merchant's receipts for several weeks. Find his total receipts for the Mondays, the Tuesdays, the Wednesdays, the Thursdays, the Fridays, and the Saturdays; also the total receipts for each week and for the whole time.

Week	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
1st.....	$2,564·15	$1,575·73	$1,026·81	$4,830·65	$1,899·73	$1,675·00
2nd.....	584·62	873·15	860·43	873·14	764·43	1,742·15
3rd.....	2,542·76	1,206·72	1,206·72	3,210·95	1,568·70	1,672·19
4th.....	3,782·00	923·84	923·84	972·67	1,097·35	638·25
5th.....	1,345·75	2,437·96	2,437·96	2,056·80	587·76	1,253·08

30. What is the difference between his total receipts for the first and second weeks, and the total for the third and fifth weeks?

31. How can you prove the correctness of the number which you gave as the grand total of the receipts?

32. Which of the digits in a period gives the period its name? What word means the reading or writing in words of a number already expressed in figures? What name is given to the expression of numbers in figures?

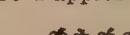
33. What will be the total cost of 4 bushels of wheat at 98 cents a bushel, 5 bushels of oats at 45 cents a bushel, and 3 bushels of pease at 75 cents a bushel?

34. What two names are used for the symbols in Arabic notation? Give, beginning at the right-hand side of a number, the names of the first three periods.

35. In six days, the receipts from a street railway were $5,123·25. The expenses were $2,495 for wages, and $629·89 for repairs. Find the profits.

MULTIPLICATION

How many apples are 4 apples + 4 apples + 4 apples?



Four apples repeated 3 times are 12 apples, or we may express this thus: 3 times 4 apples are 12 apples.

Two times, or twice, 3 pears are how many pears? Five times 2 oranges are how many? Seven times 3 plums? Six times 3 peaches?

The expression 5×3 is read 3 times 5, and means $5 + 5 + 5$ or 5 taken 3 times as an *addend.*

Express each of the following, using both the sign $\times$ and the sign $+$, thus 3 times $5 = 5 \times 3 = 5 + 5 + 5$:

4 times 4.	2 times 7.	3 times 7.
3 times 6.	5 times 6.	4 times 8.

Find, by adding, how many are 3 times 5; 4 times 9; 6 times 4.

Multiplication is finding, by using memorized results, the sum obtained by repeating one number as an addend as many times as there are units in another number.

The number repeated or multiplied is called the **multiplicand.** The number that shows how many times the *multiplicand* is to be repeated as an addend is called the **multiplier.** The result got by multiplying is called the **product.**

Such expressions as 14×4, 12×6, 10×3 are read 14 multiplied by 4, 12 multiplied by 6, 10 multiplied by 3.

Read each of the expressions 13×6, 18×9, 16×7, in two ways.

What does the *multiplier* tell us about the multiplicand? How is the *product* found? In the question, 15 apples $\times 7$, what must the name of the product be? If 115 dollars

is multiplied by 9, what will be the name of the product? What terms in *multiplication* must have the same name? why?

Multiplication is a short method of performing what other operation?

Which number in multiplication corresponds to the sum in addition? Which corresponds to the addend? What, in addition, corresponds to the multiplier?

Find by addition:

2 times 1, 2 times 2, and so on to 2 times 12;
3 times 1, 3 times 2, and so on to 3 times 12.

In this way, you will form the **multiplication tables** of 2 times and 3 times.

In the same way, make the tables of 4 times, 6 times, 7 times, 8 times, and 9 times, putting the right figures in the blank spaces in the following form:

1	2	3	4	5	6	7	8	9
2	4	6						
3	6	9						
4	8	12						
5	10	15						
6	12	18						
7	14	21						
8	16	24						
9	18	27						
10	20	30						
11	22	33						
12	24	36						

This completed table should be thoroughly committed to memory.

How would you multiply 483 by 6?

$$\begin{aligned} 483 &= 4 \text{ hundred} + 8 \text{ tens} + 3 \text{ units.} \\ 483 \times 6 &= 4 \text{ hundred} \times 6, + 8 \text{ tens} \times 6, + 3 \text{ units} \times 6 \\ &= 24 \text{ hundred} + 48 \text{ tens} + 18 \text{ units} \\ &= 2400 + 480 + 18 = 2898. \end{aligned}$$

Time may be saved by arranging the work thus:

$$\begin{array}{r} 483 \\ \underline{\quad 6} \\ 18 = 3 \text{ units} \times 6 \\ 480 = 8 \text{ tens} \times 6 \\ \underline{2400 = 4 \text{ hundreds} \times 6} \\ 2898 = 483 \times 6. \end{array}$$

We can still further save time in this way:

$$\begin{array}{r} 483 \\ \underline{\quad 6} \\ 2898 \end{array}$$

6 times 3 units make 18 units, or 1 ten and 8 units. Place the 8 in the units' place and reserve the 1 ten. Then 6 times 8 tens make 48 tens. This with the 1 ten which we reserved will make 49 tens or 4 hundred and 9 tens. Put the 9 in the tens' place and keep the 4 hundred to add to the next product. Again, 6 times 4 hundred make 24 hundred. This with the 4 hundred reserved will make 28 hundred or 2 thousand 8 hundred.

EXERCISE 14

1. Multiply 6893 by 2, by 4, by 6, and by 8.

2. Multiply 47156 by 3, by 5, by 7, and by 9.

3. Find the product of 57092 by 2, by 6, and by 9.

4. Multiply 309758 by 3, by 4, by 5, by 7, and by 9.

5. Find the product of eighty-seven thousand four hundred and six by 4, by 8, and by 9.

6. Make the multiplication table of 10 times, then of 11 times, and then of 12 times in the same way that the

tables of 2, 3, 4, 5, etc., times were made. Commit these tables to memory.

7. Multiply four million eighty-nine thousand eight hundred and fifty-seven by 3, by 4, by 5, by 6, by 7, and by 9.

8. Multiply 7508794 by 8, by 7, and by 12.

9. Multiply 83009 by 9, by 7, and by 6.

10. Multiply 9 by 10, 8 by 10, 12 by 10, 24 by 10, 398 by 10.

$9 \times 10 = 90$
$8 \times 10 = 80$
$12 \times 10 = 120$
$24 \times 10 = 240$
$398 \times 10 = 3980$

Comparing, in each case, the multiplicand with the product you notice that the product is the multiplicand with a naught, or cipher, placed to its right.

11. Multiply each of the following numbers by 10: 93789, 408537, 39207.

Numbers which when multiplied together make another number are said to be **factors** of that number. Thus 6 and 4 are factors of 24; 2 and 15 are factors of 30; 3, 2, and 6 are factors of 36. Why? Name two factors of 12; of 15; of 18; of 32. What are the factors of the **product** in multiplication?

Four is one factor of 20, what is the other? Six is one factor of 30, what is the other?

EXERCISE 15

1. Name two factors of 8. Multiply 36 by 8. Then multiply 36 by one of the factors of 8 and the product by the other factor:

(*a*) $36 \times 8 = 288$. $36 \times 4 = 144$. $144 \times 2 = 288$.

Or (*b*) $36 \times 4 \times 2 = 288$.

In the same way multiply:

(*c*) $29 \times 9 = 261$. $29 \times 3 = 87$. $87 \times 3 = 261$.

(*d*) $29 \times 3 \times 3 = 261$.

Compare the result in (*a*) with the result in (*b*) and then the result in (*c*) with the result in (*d*).

2. 8 times 36 is how many times 4 times 36? 9 times 29 is how many times 3 times 29? 14 times 41 is how many times 7 times 41? 24 times 347 is how many times 6 times 347?

3. What is the difference between 347×12 and $347 \times 6 \times 2$?

4. Multiply 3876 by 10 and then multiply it by 5 and 2 (the factors of 10). Compare the two results.

5. Multiply 58,093 by 24, 25, 28, 32, 36, 44, and 49 separately, using factors.

6. In the same way, find the product of ninety-seven thousand four hundred and sixty-eight and 54, 55, 56, 72, 77, 81, and 84 respectively.

7. Multiply 364289 by 90, 96, 99, 108, 120, 121, 132, 144 separately. In each case use factors.

8. Using factors, multiply 894 by 100, 832 by 100, 70096 by 100. In each case compare the product with the multiplicand: 894 with 89400, 832 with 83200, and 70096 with 7009600.

9. In what very short way can any number be multiplied by 100?

10. What two numbers are factors of 1000? Multiply 347 by 1000, using the factors 100 and 10.

11. Give a short way of multiplying by 1000.

12. Without actually multiplying, put down the product of 3475 and 1000, of 45679 and 1000, of 900750 and 10000.

What number, with 10, is a factor of 20?

Twenty times 279 is how many times 2 times 279? How many times 3 times is 30 times? How many times 4 times is 40 times?

If you have 2 times a number, how can you easily get 20 times the same number? If you have 3 times 49, how can you easily get 30 times that number?

EXERCISE 16

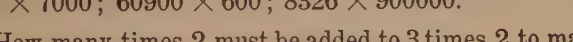
1. Give a short way of multiplying by 20, by 30, by 40, by 50.

2. Multiply 45678 by 20, then by 30, then by 40, then by 50. Use the shortest method you can.

3. Multiply, in the shortest way, 6794 by 200, then by 300, then by 3000, then by 800.

4. Find in the shortest way, the products: 4070×400, 67024×7000; 60900×600; 8326×900000.

5. How many times 2 must be added to 3 times 2 to make 5 times 2?



6. How many times 3 must be added to 2 times 3 to make 6 times 3? Use dots to find this out. How many times 4 must be added to 3 times 4 to make 7 times 4?

7. How many times 18 must be added to 3 times 18 to make 9 times 18?

8. Find 9 times 18 in as many ways as you can. Find 4 times 275 in 3 ways. If ten times 25 is added to 2 times 25, the sum will be how many times 25? This is generally done this way:

$$\begin{array}{r} 25 \\ 12 \\ \hline 50 = \text{ 2 times } 25 \\ 250 = 10 \text{ times } 25 \\ \hline 300 = 12 \text{ times } 25 \end{array}$$

EXERCISE 17

1. Using the foregoing as models, find the products:

4793×18, 6479×23, 4938×29, 732×51; 6794×67, 4912×68, 7684×69, 97854×79, 86437×89, 26089×59.

2. Looking at 1402 you will see that it is the sum of 3 numbers. What are they?

3. Look carefully at 40012. Of what 3 numbers is it composed?

In multiplying 9876 by 6789 the work is put down thus:

9876	
6789	
88884	Each of the four results which added together make the product is said to be a **partial product.**
790080	
6913200	
59256000	
67048164	

4. Multiply 76841 by 14002, 2397 by 40012, 7901 by 3601, 8496 by 3728, 40571 by 8070, 12035 by 60018, 7707 by 6906, 65432 by 50701, 40876 by 865, 23041 by 786, 92842 by 493.

5. By what 3 numbers did you multiply the first multiplicand? the second? the third? by what two the fifth? by what three the seventh?

6. The multiplier is nine thousand and eight and the multiplicand sixty thousand four hundred and seventy-nine. Find the product.

7. Compare the products of 357×843 and 843×357; and of 459×738 and 738×459.

8. You have already learned that the multiplicand and multiplier are factors of the product. To get the product of any two numbers, does it make any difference which number is taken as multiplier?

NOTE: Proving the correctness of any calculation is merely doing the work again, but in some other way.

9. Examine Question 7 again very carefully, and then say how the accuracy of work in multiplication may be proved.

10. Find the products and prove their correctness:

89756 × 978, 86405 × 700732, 76841 × 3760000, 837621 × 89795, 987701 × 56789, 398764 × 476975.

11. If, in the first Question in 10, you, by mistake, multiplied 89756 by 980 instead of by 978, your product would be too large. How many times the multiplicand is it too large?

If 2 times the multiplicand be taken from the incorrect product, will the right product be obtained?

12. Multiply 35,976 by 399 and then by 400. What should be done to the second product to make it equal to the first?

13. Multiply 7489 by 150 and then by 200. How many times the multiplicand must be taken from the second product to make it the same as the first?

14. Multiply 459 by 942 and then by 940. What is needed to make the second product the same as the first?

15. Multiply and prove your answers to be correct either by multiplying by factors, by multiplication and subtraction, or by multiplication and addition, being careful to use the shortest and easiest kind of proof: 432 by 36, 479 by 32, 581 by 52, 953 by 56, 8647 by 365, 5927 by 395, 9467 by 769, 8304 by 9782.

In the illustration following Question 3 of this Exercise the partial products may be put down thus:

```
  9876
  6789
 -----
 88884
79008
69132
59256
-------
67048164
```

How many times the multiplicand would the sum of the first and second partial products be? the sum of the second and third? the sum of the first and third? the sum of the first and last? the difference between the third and fourth?

If you examine the eight numbers, 5, 5 cows; 6, 6 sheep; 9, 9 dollars; 12, 12 inches; you will readily see that 4 of

them (5 cows, 6 sheep, 9 dollars, 12 inches) refer to some *particular* kind of thing, and that the others do not refer to any *particular* kind of thing.

A number which is used in connection with some *particular* kind of thing is called an **applied** number, or it is, by some, said to be a **concrete** number.

A number which is not used in connection with some *particular* kind of thing is called an **abstract** number.

Can a number be repeated 6 dollars times? 8 cows times? 5 yards times?

Then can a multiplier be an *applied* number? What kind of a number must it be?

Name 4 applied numbers and 4 abstract numbers.

EXERCISE 18

NOTE: Always *check* your work. You should be able to arrange your solution or work neatly, to state readily what each step in it is, and to give a good reason for it. Nothing short of accuracy is commendable. An accountant who makes 5 errors in 100 is not reliable.

1. An office desk costs $25. How much will 3 such desks cost? 8 desks? 36 desks? 49 desks?

2. Eggs sell for 23c. a dozen. Find the cost of 8 doz., 18 doz., 94 doz.

3. There are 5280 feet in a mile. How many feet are there in 19 miles? in 76 miles? How may the second answer be found from the first?

4. How many days in 39 weeks? in 214 school weeks? in 809 working weeks?

5. How many ounces in 1 pound? in 169 pounds? in 144 pounds and 12 ounces?

6. How many inches in 769 feet? How many days in 17 common years?

7. The rent of a dwelling is $28 a month. Find the rent for three years.

8. A carpenter earns $3·20 a day. At this rate, how much wages will he receive in 298 days ?

NOTE: In solving Question 8, will it save time to use 300 instead of 298 and then take twice $3·20 from the product ?

9. If $630 will keep a family in provisions for 8 months, what will be required to keep the same family for 72 months ?

10. If 12 men can cut a pile of wood in 24 days, in how many days can one man do the same work ?

11. At a sale there were sold 9 pigs at $8 each, 7 sheep at $6 each, 9 horses at $165 each, and 12 cows at $45 each. How much was received for all ?

12. Two men bought farms at $75 an acre. The first bought a farm of 86 acres, the second a farm of 75 acres. How much more had one to pay than the other ? *Do this in a short way.*

13. A took a railway journey of 93 miles, B travelled 9 times as far, C 12 times as far as A and B together, D 13 times as far as C less what B travelled. How many miles did all of them travel ?

14. There are 320 rods in a mile. How many rods in 479 miles and 227 rods ?

15. Light travels 185,172 miles in a second and passes from the sun to the earth in 493 seconds. What is the distance from the sun to the earth ?

16. How many pounds are there in 16 hundredweight and 14 pounds ?

17. A bushel of wheat weighs 60 pounds. How many pounds should 4876 bushels weigh ?

18. A bushel of oats weighs 34 pounds. How many pounds are there in 176 bushels ?

19. A man bought 74 bbl. of flour at $6·25 a barrel, 12 bbl. of apples at $3·50 a barrel, 65 lb. of cheese at 14c.

a pound, 54 bags of flour at $2·70 a bag, and 95 lb. of butter at 27c. a pound? Find the total cost of all his purchases.

20. Three towns are in a straight line. A is 29 miles east of Toronto, B is three times as far west of Toronto, and C is west of Toronto by 13 miles less than twice the distance from A to B. How far is it from A to C? *Make a drawing to help you in getting the answer.*

21. A farmer sold a merchant 89 bu. of potatoes at 57c. a bushel, 165 lb. of butter at 27c. a pound, and 175 doz. eggs at 34c. a dozen. In exchange, the merchant sold the farmer 13 lb. of tea at 45c. a pound, 2 bbl. of apples at $3·50 a barrel, a box of raisins at $4·27, and 24 yd. of tweed at 67c. a yard. Which person owes the other, and how much?

EXERCISE 19

1. Divide these numbers into periods and then read the numbers: 12012012, 80340910, 400897601.

2. A man earned $15 a week for 2 weeks and then did nothing for three weeks. He paid $4·25 a week for his board the whole time. How much of his earnings had he left?

3. I bought three and a half dozen eggs at 16c. a dozen and sold them at 18c. a dozen. What was my gain? *Solve this problem in a short way.*

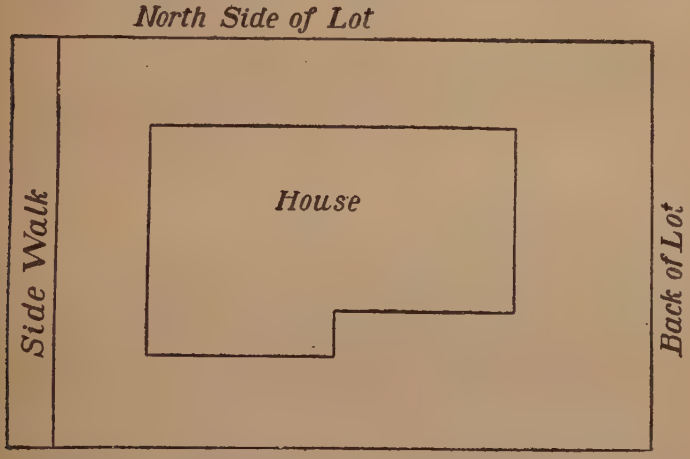
4. A drover bought 640 cattle at $48 each and 72 horses at $160 each. How much more did he pay for the cattle than for the horses?

5. A merchant had 936 yards of dress goods. He sold 253 yards to one customer and twice as much to another. What is the remainder worth at 45c. a yard?

6. What is the value of $16 \times 15 \times 19 \times 24 \times 345$?

7. Which of the three numbers in multiplication cannot be an applied number? Which must have the same names? By what other rule can we solve problems in multiplication?

8. The following diagram of a house and lot is drawn on a scale of 24 feet to an inch, that is, one inch on the figure stands for 24 feet.



Using your measuring rule, find the answers to the following questions:

(*a*) How many feet long is the lot, not including the walk?

(*b*) How many feet wide is the lot?

(*c*) How many feet from the side-walk to the house?

(*d*) How far from the house to the back of the lot?

(*e*) How far from the house to the north side of the lot?

(*f*) How many feet long is the house?

(*g*) How many feet wide is the front of the house?

(*h*) How many feet wide is the rear of the house?

(*i*) How far from the south side of the lot to the house?

(*j*) How wide is the side-walk?

(*k*) How much is the lot worth at $25 a foot-front?

NOTE: The expression "foot-front" stands for a strip 1 foot wide and as long as the lot is deep.

9. From 10th April to 1st July is how many days? From June 15th to October 4th?

10. Tickets of admission sold at County Fair:

	Price	Wed.	Thu.	Fri.	Sat.	Total
Children's tickets	15c.	1645	1154	3561	1424	
Adults' tickets	25c.	2243	1754	3871	2124	
Single carriage tickets ..	25c.	143	174	186	75	
Double carriage tickets ..	50c.	123	175	162	137	

(*a*) Find the number of tickets sold each day.

(*b*) Find the number of children's tickets sold.

(*c*) Find the number of adults' tickets sold.

(*d*) Find the number of single carriage tickets sold.

(*e*) Find the number of double carriage tickets sold.

(*f*) Find the whole number of tickets sold. How can you prove the correctness of your total?

(*g*) Find the receipts, that is, the amount *received* for tickets sold on Wednesday, (*h*) on Thursday, (*i*) on Friday, (*k*) on Saturday, (*l*) for all the tickets sold.

(*m*) Find the receipts for all the children's tickets sold during the week, (*n*) for adults' tickets, (*o*) for single carriage tickets, (*p*) for double carriage tickets.

(*q*) Find the sum of (*m*), (*n*), (*o*), (*p*), and compare this sum with the answer to Question (*l*).

11. A man bought 7 sheep at $11 a head, twice as many cows each at 3 times the price of a sheep, 4 times as many horses as cows each at 5 times the price of a cow, and enough steers to make 100 animals in all. Each steer cost the difference between the price of a sheep and the price of a cow. How much did he pay for all?

12. Solve each of the following, and make a business problem suggested by it:

(*a*) Multiply \$475 by 9. (*b*) Multiply 3759 by 47.
(*c*) Multiply \$6789 by 847. (*d*) Multiply 98765 by 3908.

13. During the month of November, 1898, there were consumed at a college in Ontario 64 loads of coal. The weight of each load, in pounds, is given below. Find the total weight:

6100	8020	5490	5190
8380	6860	6800	7130
4850	6230	6560	7090
8010	6780	6690	7790
7080	6980	5780	6810
6620	6240	6980	8600
6450	6310	5990	9100
6570	6300	4740	6740
7950	6530	5520	5380
4750	6950	3630	7640
8840	6980	4930	5650
7290	4920	5150	5900
4960	5880	6420	6200
8330	7030	6770	6620
6300	6420	6220	7170
7080	5160	6020	9210

14. Multiply 98276 by 789, 6789 by 6764, 78462 by 807, 3279 by 9865.

15. Name two factors the product of which is 24, is 18, is 36, is 42, is 64, is 108.

Numbers which can be divided into *factors* are said to be **composite** numbers.

Numbers which cannot be resolved into *factors* are called **prime** numbers.

16. Write all the prime numbers from 1 to 29 inclusive.

17. Write all the composite numbers from 30 to 72 inclusive.

DIVISION

ORAL EXERCISE

1. If 24 dollars are divided equally among 6 boys, how many dollars will each get? What is the *sixth* part of 24?

2. If 30 oranges are divided equally among 5 boys, how many oranges will each boy get? What is the *fifth* part of 30?

3. If 54 cents are divided equally among nine girls, how many cents will each get? What is the *ninth* part of 54?

4. If a boy can walk 3 miles an hour, how many hours will it take him to walk 12 miles? How many 3's are in 12?

5. If a boy can ride 7 miles an hour on a bicycle, how many hours will it take him to ride 21 miles? How many 7's are in 21?

6. If 56 dollars are divided among a number of boys so that each boy receives 7 dollars, how many boys are there? How many 7's are in 56?

7. If a cord of wood costs $5, how many cords can be bought for $23? How many 5's are in 23?

8. If a pound of sugar costs 6c., how many pounds can be bought for 25c.? How often is 6 contained in 25?

9. How often is 9 contained in 75? How many are left over?

In this example, 9 is called the **divisor,** 75 is called the **dividend,** 8 is called the **quotient,** and 3 is called the **remainder.**

NOTE: Division is (1) the process of finding how many times one number is contained in another, or (2) it is finding one of the equal parts of a number.

10. How much is 10 times $17? How many times is $17 contained in $170?

11. How many times is $3 contained in $12? in $15? in $21? in $42?

12. If the product of two numbers is 35 feet and one of the numbers is 7 feet, what is the other? If one of the numbers is 5, what is the other?

From these examples the relation between multiplication and division may easily be seen. In multiplication *two factors* are given, and their *product* is required. In division, on the other hand, the *product* and *one factor* are given, and the *other factor* is to be found.

In division, which of the four terms, *divisor*, *dividend*, *quotient*, or *remainder*, corresponds to the *product* in multiplication? which to the *multiplicand?* which to the *multiplier?* Give reasons for your last two answers.

Since the problem, "How many times is 7 contained in 28?" may be stated thus: "How often can 7 be taken away from 28?" by what process besides division may the problem be solved?

Which of the numbers in subtraction corresponds to the *dividend?* which to the *divisor?* which to the *quotient?*

Division is indicated by writing the sign ÷ between the numbers; thus, $8 \div 2$ means 8 divided by 2. It is also indicated by writing the dividend above and the divisor below a horizontal line; thus, $\frac{8}{2}$.

Because 10 times \$17=\$170, therefore \$170÷\$17=10, and \$170÷10=\$17.

From these problems it may be seen:

(1) That if the dividend and divisor are applied numbers, they must be *alike*, that is, they must be of the same kind, and the quotient must be *abstract.*

(2) That if the dividend is an *applied* number and the divisor *abstract*, the quotient is like the dividend.

If the combinations in multiplication have been mastered, the results in the following examples may be named rapidly.

ORAL EXERCISE

Below, the product and one factor are given, name the other. For instance, 9 is one factor of 27, what is the other factor?

9 ?, 27;	14, 7 ?, ?, 28;	15, 10, 6 ?, ?, ?, 30;	16, 8 ?, ?, 32;
17 ?, 34;	7 ?, 35;	18, 12, 9, 6 ?, ?, ?, ?, 36;	19 ?, 38;
13 ?, 39;	20, 10, 8 ?, ?, ?, 40;	21, 14, 7 ?, ?, ?, 42;	22, 11 ?, ?, 44;
15, 9 ?, ?, 45;	23 ?, 46;	24, 16, 12, 8 ?, ?, ?, ?, 48;	7 ?, 49;
25, 10 ?, ?, 50;	17 ?, 51;	26, 13 ?, ?, 52;	11 ?, 55;
27, 18, 9 ?, ?, ?, 54;	28, 14, 8 ?, ?, ?, 56;	19 ?, 57;	29 ?, 58;

48, 32, 24, 16, 12
?, ?, ?, ?, ?, 96;

45, 30, 18, 15, 10
?, ?, ?, ?, ?, 90.

NOTE: Other exercises similar to the above should be solved.

ORAL EXERCISE

State the quotients rapidly:

1. 144, 48, 96, 36, 120, 72, 24, 132, 84, 108, 60÷12.
2. 99, 27, 54, 90, 108, 18, 63, 81, 36, 72, 45÷9.
3. 72, 48, 24, 96, 16, 80, 40, 32, 88, 56, 64÷8.
4. 70, 14, 49, 63, 77, 56, 28, 84, 21, 42, 35÷7.
5. 24, 42, 72, 54, 36, 60, 12, 66, 30, 18, 48÷6.
6. 50, 10, 20, 30, 40, 60, 45, 55, 35, 15, 25÷5.
7. 44, 28, 20, 48, 12, 32, 40, 36, 16, 24, 8÷4.
8. 27, 18, 6, 36, 24, 9, 33, 21, 12, 30, 15÷3.

Name the quotients rapidly:

9. 98÷49, 98÷14, 96÷24, 96÷16, 95÷19, 94÷47, 92÷23, 91÷13, 90÷18, 90÷30, 88÷22.

10. 87÷29, 86÷43, 84÷28, 84÷14, 82÷41, 80÷16, 78÷13, 76÷19, 75÷15, 74÷37, 72÷24, 72÷36, 72÷18.

11. 70÷14, 68÷17, 66÷22, 65÷13, 64÷16, 63÷21, 62÷31, 60÷15, 58÷29, 57÷19, 56÷14, 54÷18.

12. 52÷13, 51÷17, 48÷16, 48÷24, 46÷23, 45÷15, 42÷14, 39÷13, 38÷19, 34÷17, 32÷16, 28÷14.

13. 24÷12, 34÷17, 48÷8, 27÷9, 35÷7, 39÷13, 45÷9, 26÷13, 42÷14, 60÷12, 44÷11, 33÷11, 42÷21, 32÷16, 42÷21, 45÷15, 84÷12, 84÷21, 84÷42, 96÷12, 36÷18, 65÷5, 66÷11, 68÷4.

14. 72÷18, 38÷19, 30÷15, 75÷5, 75÷15, 90÷18, 68÷17, 95÷19, 51÷17, 48÷16, 84÷6, 98÷14.

The Remainder.—The dividend is not always the product of the divisor and a whole number or integer. Thus 9÷4=2 and 1 left; 37÷5=7 and 2 left. The number left over is called the **remainder** and is usually written above, and the divisor below, a horizontal line to show that the division has not been performed. Thus 89÷9=9 and 8 as a remainder. The remainder is written $\frac{8}{9}$ to indicate that it is undivided. The whole quotient is written $9\frac{8}{9}$.

ORAL EXERCISE

1. 19÷3, 17÷4, 20÷3, 21÷2, 11÷3, 17÷2, 26÷5, 29÷3, 33÷4, 27÷6, 43÷7, 31÷8, 15÷7, 32÷7.

2. 16÷6, 27÷4, 37÷4, 29÷3, 39÷2, 38÷3, 19÷8, 43÷8, 26÷3, 35÷6, 37÷3, 29÷7, 41÷5, 27÷5.

3. 22÷5, 38÷8, 46÷5, 36÷7, 48÷7, 41÷6, 49÷6, 33÷7, 42÷5, 36÷7, 51÷5, 47÷9, 53÷7, 42÷9.

4. 13÷6, 26÷7, 34÷4, 36÷5, 49÷3, 51÷2, 43÷3, 46÷7, 61÷3, 22÷7, 46÷3, 28÷8, 39÷7, 51÷7.

Find the value of:

5. 119, 117, 111, 113, 14, 17, 86, 118, 90, 109, 91÷12.
6. 65, 97, 45, 110, 78, 56, 32, 75, 23, 85, 98÷12.
7. 107, 68, 73, 95, 25, 59, 64, 28, 35, 102, 80÷11.
8. 20, 50, 40, 70, 100, 60, 30, 90, 85, 63, 79÷11.
9. 89, 58, 26, 67, 39, 76, 16, 32, 48, 83, 60÷9.
10. 15, 34, 55, 70, 80, 20, 42, 64, 86, 82, 75÷9.

A whole may be separated into **equal parts.** When a whole is separated into 2 equal parts each part is a *half;* when a whole is divided into 3 equal parts each part is a *third;* when a whole is divided into 8 equal parts each part is an *eighth;* and so on.

Dividing by 2 is finding a *half,* dividing by 3 is finding a *third,* dividing by 5 is finding a *fifth,* and so on.

ORAL EXERCISE

1. Find the value of one part when 12 cents is divided into 4 equal parts; when 18 cents is divided into 3 equal parts; 20 cents into 5 equal parts; 28 cents into 4 equal parts; 45 cents into 9 equal parts.

Find the value of:

2. 1 half of \$6, 1 third of \$12, 1 fourth of \$20, 1 fifth of \$30, 1 sixth of \$12, 1 seventh of \$56, 1 ninth of \$45, 1 eighth of \$72.

3. 1 ninth of \$81, 1 fourth of \$56, 1 eighth of 96 cents, 1 ninth of 72 cents, 1 fourth of 80 cents, 1 thirteenth of 91 cents.

4. 1 seventh of 84 cents, 1 eighth of 64 cents, 1 ninth of 90 cents, 1 eighth of 56 cents.

A whole may contain a part an exact number of times.

A whole contains its half two times. A whole contains its third three times. A whole contains its fifth how many times? its tenth how many times?

5. How many times does 26 days contain 13 days? 24 days contain 12 days? 20 hours contain 2 hours? 16 hours contain 4 hours? 96 gallons contain 6 gallons? 60 quarts contain 4 quarts?

6. How many times is 2 pints contained in 18 pints? 3 pints in 12 pints? 4 pecks in 20 pecks? 5 pecks in 30 pecks? 3 pounds in 15 pounds? 6 pounds in 72 pounds? 5 ounces in 30 ounces? 7 ounces in 42 ounces?

NOTE: The examples on pages 40, 41 and 42 should be reviewed as follows: Ex. 1, page 40: 144 contains 12 twelve times, 48 contains 12 four times. Ex. 14, page 41: 72 contains 18 four times, 38 contains 19 two times. Ex. 10, page 42: 15 contains 9 one time with 6 remaining, 34 contains 9 three times with 7 remaining.

SHORT DIVISION

When the divisor is not greater than 12 we generally use what is called **short division,** and when the divisor is greater than 12, what is called **long division.**

Find the quotient of 8619 divided by 12.

$$12)8619$$
$$718\tfrac{3}{12}$$

Explanation 1: One twelfth of 86 hundred is 7 hundred with a remainder of 2 hundred; 2 hundred is equal to 20 tens; 20 tens and 1 ten are 21 tens; one twelfth of 21 tens is equal to 1 ten with a remainder of 9 tens; 9 tens are equal to 90 units; 90 units and 9 units are equal to 99 units; one twelfth of 99 units is equal to 8 units and 3 units remaining. Hence one twelfth of 8619 is 718 with a remainder of 3. To indicate, or show, that this 3 is undivided we write it thus: $\frac{3}{12}$. The whole quotient is, then, $718\frac{3}{12}$. It is usual to call 718 the quotient and 3 the remainder.

Explanation 2: Twelve is contained in 86 hundred 7 hundred times with a remainder of 2 hundred; 2 hundred equals 20 tens; 20 tens and 1 ten make 21 tens. 12 is contained 1 ten times in 21 tens with a remainder of 9 tens; 9 tens and 9 units equal 99 units. 12 is contained 8 times in 99 units with a remainder of 3 units. Hence, 12 is contained $718\frac{3}{12}$ times in 8619.

How may division be proved? If there is a remainder, how may division be proved?

EXERCISE 20

Divide and prove, explaining in full:

1. 2)864
2. 3)981
3. 9)648
4. 8)1238
5. 9)369
6. 11)858
7. 7)504
8. 6)3255
9. 4)2367
10. 3)4163
11. 9)718
12. 7)623
13. 12)7000
14. 4)916
15. 3)7070
16. 2)5007

EXERCISE 21

(FOR ORAL AND WRITTEN WORK)

For oral practice, name each figure of the quotient without writing it:

1. 12)567024
2. 9)3063205
3. 11)30670508
4. 9)1023456789
5. 11)781605
6. 7)36430366123
7. 9)20345607
8. 8)3321456648
9. 9)6120561367
10. 5)3245321
11. 8)72867408
12. 7)1111111111
13. 7)333333
14. 7)300000051230
15. 6)7340962416
16. 12)87654321
17. 11)5443322344
18. 8)2020607325
19. 9)3245013546

ORAL EXERCISE

1. At 50c. a pound how many pounds of tea can be bought for: \$2? \$4? \$8? \$26? \$12½? \$20·50?

2. At 10c. each how many boxes of sardines can you get for: $2? $3? $10? $23? $9·30? $12?

3. At 12c. each how many cans of tomatoes can I buy for: 80c.? $1? $2? $3·20? $4·60? $5·80?

4. Find the cost of 1 doz. handkerchiefs at 25c. each.

5. If you can get 16 lb. of sugar for $1, how many pounds can you get for $5½?

6. At 50c. each how many penknives can be bought for $10·50?

7. I bought 4 melons at 25c. each, a bunch of celery at 10c., and a head of cabbage for 10c. I paid with a two-dollar bill. What was my change?

8. How many boxes, holding 8 ounces each, will be needed to hold 10 pounds of candy?

9. I bought 2 pounds of honey at 15c. a pound, and 6 bars of soap at 6c. a bar. I gave the clerk a fifty-cent piece and a twenty-five-cent piece. What was my change?

10. Subtract $4·36 from $5. Subtract $8·90 from $10.

11. How much will 1½ pounds of butter cost at 24c. a pound?

12. A boy bought apples at the rate of 3 for 2 cents, and sold them at the rate of 2 for 3 cents. How much profit did he make on 30 apples?

13. How many bushels of wheat at 90 cents a bushel can be bought for $2·70?

14. At 24c. a gallon how many gallons of coal oil will $1·44 buy?

EXERCISE 22

1. A man left an estate valued at $9000. Of this, $4000 was to be given to his widow, and the remainder to be divided equally among his 4 children. Find each child's share?

2. How many pints are there in a gallon? How many gallons in 296 pints?

3. How many feet in 300 inches? in 1632 inches?

4. How many bushels in 132 pecks?

5. On a bicycle, a man rides 9 miles an hour. How many hours will it take him to go 261 miles? After riding 19 hours, how far from the end of his journey will he be?

6. A teacher receives a salary at the rate of 4 dollars a day for every day he teaches. His yearly salary is $848. How many days are in the school year?

7. How many weeks are there in 364 days?

8. By buying horses at $75 each and selling them at $84 each a dealer makes a profit of 324 dollars. How many does he sell?

9. Divide 27 reams 18 quires of writing paper into 3 equal parts. Note: 20 quires = 1 ream.

10. How many feet of wire-fencing will it take to reach around my flower plot which is 8 yards long and 3 yards wide?

11. How much must be added to the sum of 24 and 25 to give the product of 48 and 12?

12. A farmer mixes 3 bushels of oats worth 28 cents a bushel with 5 bushels of barley worth 36 cents a bushel. What will 1 bushel of this mixture be worth? 3 bushels?

13. It is found that if 497 be subtracted 9 times from a certain number the remainder is 200. Find the number from which the subtraction has been made.

14. How many 7's must be added together to get 819?

15. How often can we subtract 9 from 210,015?

16. If a pea vine grows 2 inches in a day, how many feet will it grow in 3 weeks?

17. How many days are there in all the months which have no *r's* in their names?

18. How many toes have 19 cats?

19. From what number must 309 be subtracted 5 times to leave 173?

20. If you have 780 inches of wire, into how many foot-pieces can you cut it?

21. Draw a line one rod ($16\frac{1}{2}$ feet) long on the floor. Make marks on it to indicate feet and yards. How many yards in a rod? how many inches?

22. How many nails does it take to fasten a horse's shoe? How many horse-shoes can be fastened with 864 nails? How many horses can be shod "all around" with this number of nails?

LONG DIVISION

When the dividend is a large number and the divisor is greater than 12, it is usual to show all the steps of the process. This is called *long division.*

The following examples illustrate or show the process:

Example 1: Divide 135,943 by 37.

```
     4
    70
   600
  3000
37)135943
  111000 = 3000 × 37
  ------
   24943 still to be divided
   22200 = 600 × 37
  ------
    2743 still to be divided
    2590 = 70 × 37
  ------
     153 still to be divided
     148 = 4 × 37
  ------
       5 still to be divided.
```

Explanation: Beginning at the left of the dividend 1 hundred thousand ÷ 37 equals no hundreds of thousands; 13 tens of thousands ÷ 37 equals no tens of thousands; but 135 thousands ÷ 37 = about 3 thousand. We place this 3 above the thousands' figure of the dividend.

Subtracting 3000 times 37 from the dividend we have 24943 still to be divided. If we divide 24 thousands by 37 we shall get 0 thousands, but if we divide 249 hundreds by 37 we

shall get about 6 hundred. This 6 we place over the hundreds' figure of the dividend. Subtracting 600 times 37 from 24943 there still remains 2743 to be divided. $2743 \div 37$ equals about 7 tens (70), which we put in the right place over the dividend. Then subtracting 70 times 37 from 2743 there still remains 153 to be divided. $153 \div 37$ equals about 4, which we place above the units' digit of the dividend. As this 5 cannot be divided by 37 we must be satisfied by merely indicating the division, that is, showing that it is to be divided. This, as you already know, is done thus: $\frac{5}{37}$. The whole quotient is, then, $3674\frac{5}{37}$.

It may be written over the dividend thus:

$$\begin{array}{r} 3674\frac{5}{37} \\ 37)135943 \end{array}$$

It is sometimes written thus:

$$37)135943(3674\tfrac{5}{37}$$

Example 2: Divide 4327658 by 3754.

$$\begin{array}{r} 1152\frac{3050}{3754} \\ 3754)4327658 \\ 3754 \\ \hline 5736 \\ 3754 \\ \hline 19825 \\ 18770 \\ \hline 10558 \\ 7508 \\ \hline 3050 \end{array}$$

ORAL EXERCISE

1. If, when the work of dividing one number by another has been completed, there is no remainder, what are the factors of the dividend?

2. The quotient is 6 and the dividend 144. What is the divisor?

3. The divisor is 8, the quotient is 10, and the remainder 5. What is the dividend ?

4. If the dividend is 137, the quotient 12, and the remainder 5, what is the divisor ?

5. Three times a certain number and 1 more would be 13. What is the number ?

6. Twice my age and 3 years more would be 43 years. How old am I ?

7. When there is no remainder, how may you prove that your solution of a problem in division is correct ? When there is a remainder, how may you prove it ?

EXERCISE 23

Divide, and prove your work :

1. 24384 by 48.
2. 16686 by 54.
3. 32199 by 57.
4. 8070896 by 27.
5. 3860945 by 604.
6. 7096034 by 390.
7. 8634278 by 634.
8. 7046650 by 782.
9. 9720596 by 378.
10. 1096065 by 308.
11. 2768453 by 8307.
12. 7825893 by 247.
13. 24178012 by 327.
14. 680096 by 72.
15. 430765 by 68.
16. 568374 by 59.
17. 378096 by 88.
18. 60217658 by 601.
19. 276401234 by 3586.
20. 276301786 by 2761.
21. 6192138 by 1653.
22. 98764801 by 1976.
23. 3247653921 by 98632.
24. 910008100 by 3778.
25. 360172486 by 56794.
26. 2486012016 by 97640.

ORAL EXERCISE

Divide :

1. (*a*) 288 by 144.
   (*b*) 144 by 72.
   (*c*) 96 by 48.
   (*d*) 48 by 24.
   (*e*) 4 by 2.

2. In the preceding problems what was done to the dividend and divisor in (*a*) to get the dividend and divisor

in (*b*)? What was done to the terms in (*a*) to make those in (*c*)? What was done to the terms in (*c*) to make those in (*d*)? What to the terms in (*d*) to make those in (*e*)?

3. How do the five quotients compare with each other?

4. If the dividend and divisor be divided by the same number, what change will be made in the quotient?

5. If the divisor and dividend be both multiplied by the same number, how will the quotient be changed.

6. Divide each of the numbers 870, 9800, 76000 by 10, by 100, and each of the last two by 1000. What short way, then, is there of dividing by 10? by 100? by 1000?

7. Making use of your answer to Question 4, how can you shorten the work of dividing 96000 by 4000? 510600 by 600? 786800 by 400?

8. Divide 6944345 by 6000.

Explanation: Strike out the ciphers in the divisor and then point off the 3 right-hand digits in the dividend. Then divide the remaining portion of the dividend by the remaining portion of the divisor.

$$6/000\overline{)6944/345}$$

$1157\frac{2345}{6000}$ quotient.

You have really divided both divisor and dividend by 1000 and, of course, the quotient is not changed.

EXERCISE 24

1. Divide 270963 by 90.

2. Divide 7486094 by 300, by 600, by 7000.

3. Divide 12345678 by 84300, by 987000.

4. A dealer sold 17 horses for \$2123·30. What was the average price at which the horses were sold?

5. A man sold 273 barrels of apples for \$745·29. Find the selling price of the apples a barrel.

6. How often is \$1·45 contained in \$36·25?

NOTE: Divisor, \$1·45 or 145c. Dividend, \$36·25 or 3625c. \$1·45 is contained in \$36·25 as many times as the number 145 is contained in the number 3625. The answer is 25 times.

7. A clothier bought a number of suits of clothes at \$15·50 a suit, and paid \$480·50. How many suits did he buy?

8. A farmer bought a farm for \$2200 paying \$27·50 an acre. How many acres were in the farm?

9. How many times is \$1·50 contained in \$10500·00? \$6·25 in \$7812·50? \$4·83 in \$9331·56?

10. Glasgow is 3240 miles from New York City. A steamer makes the voyage between these cities in 9 days. Find the steamer's average rate an hour.

11. By what number must 686 be divided to give 14 for quotient?

12. By selling sheep at a gain of \$1·50 a head a man makes a profit of \$21 in all. How many sheep does he sell?

13. How many tons of hay at \$10·50 a ton must I sell to pay for 200 bushels of wheat at 84c. a bushel?

14. How many years are there in 9125 days?

15. How many boxes of tea containing 24 lb. each, at 90c. a pound, must be given in exchange for 54 tubs of butter of 54 lb. each, at 30c. a pound?

16. Rip Van Winkle slept 20 years on the mountain. How many days did he sleep? how many hours?

17. In 30 years the population of the Dominion increased from 3,635,024 to 4,833,239. What was the average increase every year?

18. How many rods of fence will be required to inclose a lot $6\frac{1}{2}$ rods long and $4\frac{1}{2}$ rods wide? *Make a plan of the lot.*

19. What has a man earned since the 28th of last June at \$$1\frac{1}{2}$ a day, not counting the Sundays?

20. Two men, Clark and Lees, are 60 miles apart, and travel toward each other, Clark at the rate of 2 miles an hour, and Lees at the rate of 3 miles an hour. How far apart will they be in 4 hours? In how many hours after starting will they meet?

DIVISION BY FACTORS

EXERCISE 25

1. What is meant by the factors of a number? What name is given to numbers which can be resolved into factors? What are numbers which cannot be divided into factors called?

2. Name as many pairs of factors as you can, each factor being less than 13, for each of the following numbers: 18, 24, 21, 36, 63, 84.

3. (*a*) Divide 4284 by 84. (*b*) Divide the same number by 12 (one of the factors of 84), and then divide the quotient by the other factor of 84.

(*a*) $4284 \div 84 = 51$.

(*b*) $12\overline{)4284}$ $7\overline{)357}$ 51 Compare the quotient in (*a*) and the last quotient in (*b*).

4. If a number is divided by another number and also by the factors of the same divisor, will the quotients be different? Satisfy yourself that your answer is correct by dividing 1728 by 96 and then by the factors of 96. Also divide 3843 by 63 and then by the factors of 63.

5. Change 867 cents to fifty-cent pieces.

$10\overline{)867}$ $5\overline{)\ 86}$—7 17—1 Changing the cents first into ten-cent pieces, how many will there be? What is the 7 that remains? Now changing the 86 ten-cent pieces into fifty-cent pieces, how many of these are there? What is the 1 that remains?

Now in 867 cents there are 17 fifty-cent pieces. There are two remainders (1 ten-cent piece and 7 cents). How many cents are equal to these two? The whole remainder is 17 cents.

6. Examine this solution carefully:

$72\big)\begin{matrix}8)70003\\ 9)\ 8750 - 3\\ \quad 972 - 2\end{matrix}$ How many groups of 8 are there in 70003? What is the 3 which remains? 972 is the quotient obtained by dividing 70003 by what divisor? 972 is the number of groups of how many units in 70003? What does the last remainder (2) stand for?

Using factors, divide:

7. 12409 by 21, by 22, by 24, by 25.

8. 70054 by 36, by 40, by 42, by 44, by 48.

9. 4627 by 27.

10. 70654 by 105 (use 3 factors).

11. 339240 by 132 (use 4 factors).

12. Find the number of strips of carpet 2 ft. wide required to carpet a room 24, 28, or 32 ft. wide. *Make a diagram to aid in the solution.*

13. Change to yards: 384 ft., 456 ft., 723 ft., 897 ft., 5280 ft.

14. Find the number of strips of carpet 3 ft. wide required to carpet a room 27, 33, or 39 ft. wide.

15. How many strips of carpet 2 ft. wide will be needed to carpet a room 20 ft. wide? Each strip being 9 yd. long, how many yards would be needed?

16. How many yards of carpet will be needed to carpet a room 24 ft. long and 18 ft. wide, the carpet being 2 ft. wide and running along the length of the room? *Draw a plan.*

17. Multiply 4567 by 765 (*a*) beginning with the right-hand digit of the multiplier; (*b*) using the right-hand digit of the multiplier first, then the left-hand digit, and lastly the middle digit.

18. In the solution of Question 17, how many times the multiplicand will the difference between the third and second partial products be?

19. Multiply 78247 by 639 in such a way that you will have but two partial products. *639 = 630 + 9 = 70 times 9 + 9. The second partial product is how many times the multiplicand?*

20. Find the product of 832547 and 726. *Multiply in such a way that there will be only two partial products.*

21. What number must be added to 7,869,456 to make it exactly divisible by 8975?

22. Bought 360 lb. of raisins at the rate of 30 lb. for $4 and sold them at the rate of 12 lb. for $2·25. Find my profit on the transaction. Before solving the problems answer these questions: How many 30 lb. are there in 360? How many 12 lb. are there in 360?

23. Divide $286 among 4 men and 6 women, giving to each woman three times as much as to each man. If 1 share be given to each man, how many shares will all the men get? How many shares will the 6 women get? In this way, into how many shares will the whole sum be divided? Each share will be what part of the whole? How much will each share be worth? How much will all the men get? All the women? Test your solution by adding the men's and the women's shares.

24. Divide $448 among 2 men, 3 women, and 4 children, giving each man three times, and each woman twice, as much as each child.

25. Richard Scott borrows $1600 from William Bird, agreeing to pay $6 for the use of each $100 for every year until the debt is paid. The debt is not paid until the end of five years. How much will Mr. Scott have to pay for the use of the $1600? How much will it take to pay the whole debt, that is, the sum borrowed and the sum due for its use?

26. A dealer exchanged 50 loads of wheat, each containing 150 bushels, at 90c. a bushel, for 15 loads of flour at $5 a barrel. How many barrels were there in each load?

27. A man worked 221 days at the rate of $1·50 a day and took his pay in wheat at 75c. a bushel. How many bushels did he receive? *Solve in two ways.*

28. I bought a house for $15,280, paying $2,680 cash and the balance in monthly payments of $1,575 each. How many monthly payments did I make?

29. Mr. Gray leases to Mr. Byres a house for $27 a month out of which he pays for expenses $8 a month. In how many months will he gain $6,916 from the house?

30. There are 1760 yards in a mile. How many feet are there in 1 mile? How many miles in 63,360 feet?

31. After dividing as far as possible 900 acres of land into farms of 160 acres each, the owner sold what was left for $450. How much did he receive an acre? *Solve the problem mentally.*

32. Subtract the product of 375 and 25 from the product of 765 and 39 and divide the remainder by 75. What is the quotient?

CANCELLATION

What number can be broken into these factors: 48, 15, 20? What number can be broken into these: 9, 9, 7, 12?

What process is indicated when a number is placed above another and separated from it by a horizontal line?

Indicate in that way the division of

(*a*) $48 \times 15 \times 20$ by $12 \times 5 \times 4$;

(*b*) $63 \times 18 \times 42 \times 96$ by $9 \times 9 \times 7 \times 12$.

How is the quotient changed, or affected, when the divisor and dividend are both divided by the same number?

Divide 288 by 36. Express 288 by 2 factors. Express 36 by 2 factors.

Then $\frac{288}{36} = \frac{72 \times 4}{18 \times 2}$. Now divide 72, one of the factors of the dividend, and 18, one of the factors of the divisor, each by 18. Then divide 4, another of the factors of the dividend, and 2, another factor of the divisor, each by 2. You will then have $\frac{4 \times 2}{1 \times 1} = 8$, which is the same quotient as you got from $\frac{288}{36}$.

If one of the factors of the dividend and one of the factors of the divisor be divided by the same number, will the quotient be changed? If factors that are found in both

dividend and divisor, that is, that are common to both, be struck from each, will the quotient be affected? Test the truth of your answer by finding the value of $63\times18\times42\times96$ divided by $9\times9\times7\times12$ in the long way, that is, by finding the product of $63\times18\times42\times96$, then the product of $9\times9\times7\times12$, and then dividing the first product by the second.

Again $\dfrac{63\times18\times42\times96}{9\times9\times7\times12} = \dfrac{\overset{7}{\cancel{63}}\times\overset{2}{\cancel{18}}\times\overset{6}{\cancel{42}}\times\overset{8}{\cancel{96}}}{\underset{1}{\cancel{9}}\times\underset{1}{\cancel{9}}\times\underset{1}{\cancel{7}}\times\underset{1}{\cancel{12}}}$

What we really did was to divide both dividend and divisor first by 9, then by 9, then by 7, and then by 12. These operations have not changed the quotient. This easy method of division is called **cancellation.** Upon what principle does its correctness depend?

EXERCISE 26

1. By cancellation find the quotients of the following:

(*a*) $\dfrac{95\times105\times125\times150}{19\times35\times25\times50}$.

(*b*) $\dfrac{8\times48\times56\times81\times52}{78\times27\times112\times32}$.

2. In the same way find the value of $75\times146\times91\times68\times460$ divided by $26\times50\times85\times73$.

3. In the same way find the value of

(*a*) $510\times650\times216\times910$ divided by $1000\times39\times26\times72$.

(*b*) $150\times81\times48\times91\times60$ divided by $128\times27\times135\times15$.

4. If 125 bundles of hides, each containing 60 pounds, were traded for 75 barrels of oil, of 40 gallons each, worth 35c. a gallon, what were the hides worth a pound?

NOTE: Before multiplying or dividing indicate the solution as in Question 1.

In a similar manner solve these problems:

5. A jobber sold a merchant 54 sacks of rice, each containing 99 lb., and took in payment 18 barrels of beef, averaging 198 lb. a barrel, and worth 12c. a lb. What was the rice worth a pound?

6. A farmer gave a nurseryman 35 cords of wood, worth $7·50 a cord, for 15 bundles of apple trees, each bundle containing 125 trees. What did the farmer pay for each tree?

7. Find the quotient obtained by dividing the continued product of the even numbers between 11 and 21 by the continued product of the numbers 1 to 8 inclusive.

8. A tinsmith used 15 boxes of tin, each containing 72 sheets, and each sheet containing 672 square inches, to make 45 crates of pans, each crate containing 12 dozen pans. How many square inches of tin were used to make one pan?

EXERCISE 27 (REVIEW)

Check all your work. Discover new ways of doing this.

1. A has 7 loaves of bread; B, 5; C, none. The three eat all the bread, each the same amount. C pays to A and B 12c. How much should each receive? *Work this mentally.*

2. A liveryman makes an annual profit of $125 from each horse; his whole profit each year from his horses is $2,125; his horses, at first, cost $87 a head. How much did all the horses cost him?

3. If 5 horses eat 14 bu. of oats in 2 weeks, how long would it take them, at the same rate, to eat 56 bu.? *Solve this mentally.*

4. If $4·20 is paid for 3 days' work, how much will be paid for 21 days' work? At this rate for how many days' work will $8·40 pay? *Solve this mentally.*

5. If $9·00 is paid for 7 days' work, how much will be paid for 28 days' work? At this rate, for how many days' work will $45 pay?

6. Add vertically and horizontally:

$ 864·28	+ $ 87·94	+ $160·48	+ $ 28·93	+ $ 74·83	= $
15·96	+ 428·33	+ 948·55	+ 418·74	+ 583·74	=
78·80	+ 73·38	+ 59·99	+ 37·55	+ 9·69	=
3394·63	+ 684·19	+ 60·43	+ 613·84	+ 348·74	=
70·92	+ 82·96	+ 378·76	+ 584·27	+ 79·68	=
498·75	+ 758·67	+ 46·87	+ 38·76	+ 43·09	=
583·48	+ 82·38	+ 697·29	+ 8·42	+ 765·83	=
$	+ $	+ $	+ $	+ $	= $

7. Copy neatly, and complete the following bill of sale:

Cobourg, Ont., 5th Aug., 1909.

Messrs. Jones and Bro.

Bought of Amos Ham & Co.

384	bu. Oats	@ 52c.	
136	lb. Tea	@ 65c.	
327	lb. Java Coffee	@ 27c.	
36	bbl. Flour	@ $4·75	
304	bu. Corn	@ 67½c.	
2760	lb. Sugar	@ 4½c.	

8. Write in the above form a bill such as might be made out by the clerk or bookkeeper of a hardware store. Let the bill contain five items.

9. A young man spent $204 during his first term at college. This was 5 twelfths the money his father gave him for a whole year's expenses. What did his father allow him for the whole year? How many twelfths are there in a year?

10. (*a*) How many inches are there in 9 ft. 7 in.? (*b*) The wheel of a wagon 9 ft. 7 in. in circumference turns around 42 times in going from one place to another. What is the distance in inches between the two places?

11. (*a*) Find the dimensions (the length, breadth, and height) of the walls of your school-house, (*b*) of your school-room, (*c*) of each black-board.

12. If an orange is worth 3 times as much as an apple, how many times as much as the apple are both worth? If both are worth 8c., what is the apple worth? *Solve mentally.*

13. Draw a line and divide it into halves, then into thirds, and then into sixths. How many sixths are there in one half? How many sixths are there in a third?

14. A man bought 16 sheep at $3 each and sold them at the rate of 3 for $12. How much did he gain? *Solve mentally in a short way.*

15. Draw a diagram on a scale of 1 inch to 3 yards for a lot 15 yards long and 12 yards wide.

16. Copy the following accounts and find the amount due on each:

(*a*) J. Manning,

In account with D. L. Palmer, *Dr.*

1909				
Jan. 2	To 75 lb. Rice	@ $0·04		
" 2	" 330 lb. Sugar	·05		
" 7	" 50 lb. Java Coffee . .	·32		
" 7	" 45 lb. Tea	·60		
	Amount due	$		

(*b*) James Gilman,

In account with Geo. Johnson, *Dr.*

1909				
May 6	To 5 days' work . .	@ $ 2·50		
" 13	" 12 lb. Nails . . .	@ ·03		
" 14	" 7 panes of Glass .	@ ·40		
June 11	" 10 gal. Paint . .	@ 1·00		
" 18	" Job work on House .	@ 275·00		
	Amount due	$		

Port Hope, 30th Jan., 1909.

(*c*) Mr. J. Hill,
In account with F. Warner & Co., *Dr.*

Jan.	2	To	3 Chairs	@ $ 2·25	
"	9	"	1 Library Table . .	@ 25·00	
"	15	"	3 Rugs	@ 6·75	
"	20	"	40 yd. Matting . . .	@ ·45	
"	23	"	2 Wardrobes . . .	@ 17·50	
					$

Paid,
Feb. 1st, 1909.

F. W. & Co.
per W.

17. Make out bills for the following and receipt them as in the preceding:

(*a*) Daniel Jones bought of J. W. Summers, Guelph, 4th Aug., 1909, 28 yd. calico at 9¼c., 6½ yd. velvet at $2, 1 doz. linen handkerchiefs at 37½c. each, 4 pairs kid gloves at $1·50, 2½ doz. buttons at 20c., 9¾ yd. black silk at $1·40.

(*b*) Mr. Jas. Kay bought of Simpson, Perdue & Co., Napanee, 6th Aug., 1909: 50 lb. sugar at 4½c., 15 cans tomatoes at 13c., 27 cans corn at 11c., 10 packages breakfast food at 12½c., 8 cans salmon at 18c., 5 gal. maple syrup at $1·25, 25 lb. butter at 25c., 6 lb. Y. H. tea at 60c.

18. A merchant becomes bankrupt, that is, he fails in business. He owes in all $19,324. His assets, that is, everything he owns, are worth $9,662. How many cents can he pay for every dollar that he owes his creditors?

19. A man buys 400 bushels of wheat at 90c. a bushel and sells it so as to gain 25c. on every dollar he pays for it. For how much must he sell the whole of the wheat?

20. James Jamieson lends Robert Jones $960 on the understanding that the borrower is to pay the lender $8 yearly for the use of every $100 borrowed. At the end of

eight years, Mr. Jones pays the debt. How much did he have to pay altogether for the use of the money ?

21. How much does a man gain or lose on the sale of two houses which cost him $1,200 each, if he gains one third of the cost price on one and loses one fifth of the cost of the other ?

22. Add the following accurately. Try to do this in 4 minutes:

48963	85862	42187
77681	48207	63126
84476	54871	49503
66732	57532	96135
44681	39415	85674
77697	68545	76458
77536	75284	75275
54384	68452	89405
76208	46767	40606
82273	80748	56432
57642	56864	73688
72074	69589	86774

Verify your work in different ways.

23. Find the sum of all the prime numbers between 50 and 75.

24. A store-keeper bought 140 loads of potatoes, of 30 bushels each, at 37½c. a bushel. If 15 bushels were made unsaleable by frost, how much will he gain on the whole cost by selling the remainder at 17c. a peck ?

25. If 3 fifths of a piece of cloth is worth $97·50, how much is the rest of the piece worth ?

26. If 3 fourths of a ton of clover hay is worth $11·64, how much are ten tons worth ? *Work this mentally.*

27. In 9 days and 18 hours how many hours are there ? In 6 hours and 13 minutes how many minutes ?

28. Change 203 ounces to pounds and ounces. Change 4,050 minutes to hours and minutes.

29. Find answers to the following, your time limit being 4 minutes:

(*a*) $1357·64
309·37
260·03
1389·50
19·24
749·63
4·89
6009·86

(*b*) $614832·05
92876·09
Difference,

(*c*) 52962×3
4905×4
76407×6
792×7
85×9
Total,

(*d*) 1728)1264896

30. (*a*) How many pounds are there in a bushel of wheat? in a bushel of oats? in a bushel of barley?

(*b*) Make out the following bill, giving names of persons and place, and also the date. Receipt the bill.

2040 lb. of oats at 63c. a bushel; 1728 lb. barley at 71c. a bushel; 6020 lb. wheat at 90c. a bushel; 169 lb. sugar at 20 lb. for $1; 57 lb. tea at 45c. a pound.

31. Divide 320,389 by 9, then divide the quotient by 9, and repeat the process until a quotient less than 9 is reached.

32. Sold goods which cost $840 at a gain of 3 tenths of the cost. How much did I gain? What was the selling price?

33. The difference between the product of two numbers and 2,431 is three hundred million three hundred and three thousand and three. One of the numbers is 20,306. Find the other.

34. Find the product of the sum and difference of 4,569 and 3,879.

35. A farmer's wife sold 3 tubs of butter each holding 50 lb. at 25c. a pound, 24 ducks at 60c. a pair, 120 lb. lard at 14c. a pound, and 18 turkeys at $1·25 each. In payment she got $20 cash and sugar at $5 a 100 lb. How many pounds of sugar did she receive?

36. A farmer gave 125 bu. of wheat worth 80c. a bushel for 140 bu. of oats and $58 cash. What was the value of a bushel of oats?

37. A farmer's wife sold 35 doz. eggs, at 20c. a dozen, and 37 lb. 8 oz. of butter at 24c. a pound. How many pounds of tea at 40c. a pound will settle the account?

38. How many cent pieces each one inch in width can be placed in rows on a table 3 ft. 8 in. long and 2 ft. 7 in. wide? *Draw a plan.*

39. Three men, A, B, and C, together go into business. A puts $6000 into the business, B $8000, and C $4000. Their profits are $5400. By how much money was the profit made? How much profit did each dollar put into the business make? What was each man's share of the profits?

40. Upon what principle does the short method of dividing by 10, 1000, 20000, etc., depend?

41. If an applied number be divided by another applied number, what kind of number will the quotient be?

42. What kind of number (applied or abstract) must a multiplier be?

43. What must be done to the divisor to make the same change in the quotient that dividing the dividend by any number (say 8) makes?

REDUCTION

Applied numbers denoting measure, for example, $4, 3 feet, 6 ounces, 5 years, are called **denominate numbers.**

A compound denominate number is a number expressed in two or more units of the same kind. For example, 11 hours 25 minutes 15 seconds, 12 gallons 3 quarts 1 pint, 3 acres 150 square rods 25 square yards, are three compound denominate numbers.

Change 10 bushels to pecks, 8 feet to inches, 3 years to months, 6 pounds to ounces, 5 halves to sixths.

Express 960 inches as feet, 480 ounces as pounds, 3,600 minutes as hours, 272 pints as quarts, 16 eighths as fourths.

Is 10 bushels the same quantity as 40 pecks? Is 960 inches the same length as 80 feet? In the changes which have been made, were the **values** of the quantities 10 bushels, 960 inches, etc. changed?

The process of changing a number from one *denomination* or *measuring unit* to another without changing the value is called **reduction.**

Reduction to a smaller measuring unit, or denomination, is called **reduction descending.**

Reduction to a higher denomination is called **reduction ascending.**

Give two examples of *reduction descending,* and two of *reduction ascending.*

TABLES OF WEIGHTS AND MEASURES

There are three sets of measures of *weight* used in Canada, namely: Avoirdupois weight, Troy weight, Apothecaries' weight. Avoirdupois weight is the one in common use.

The Dominion standard unit of weight is the **pound** Avoirdupois.

AVOIRDUPOIS WEIGHT

16 drams (dr.) . . .	1 ounce (oz.)
16 ounces	1 pound (lb.)
100 pounds	1 hundredweight or cental (cwt.)
20 hundredweight . .	1 ton.

NOTE: 7000 grains (gr.) = 1 lb. avoirdupois. 14 lb. = 1 stone. 5760 grains = 1 lb. Troy = 12 oz. Troy.

ORAL EXERCISE

1. How many ounces in 12 lb. of lard? in 6 lb. of butter?

2. In 1200 lb. how many hundredweight? In 96 drams how many ounces? In 96 oz. how many pounds? In 84 lb. how many stone?

REDUCTION DESCENDING

Change 5 tons 15 cwt. 13 lb. 14 oz. to ounces.

(*a*)

5 tons 15 cwt. 13 lb. 14 oz.
20
100 cwt. = 5 tons
15
115 cwt. = 5 tons 15 cwt.
100
11500 lb. = 5 tons 15 cwt.
13
11513 lb. = 5 tons 15 cwt. 13 lb.
16
69078
11513
184208 oz. = 5 tons 15 cwt. 13 lb.
14
184222 oz. = 5 tons 15 cwt. 13 lb. 14 oz.

(*b*)

5 tons
20
115 cwt.
100
11513 lb.
16
69092
11513
184222 oz.

Explanation: In 5 tons how many hundredweight? In 5 tons and 15 cwt. how many hundredweight? In 115 cwt. how many pounds? In 115 cwt. and 13 lb. how many pounds? In 11513 lb. how many ounces? In 11513 lb. and 14 oz. how many ounces?

The usual way of arranging the work is shown in (*b*), the additions being made when multiplying.

REDUCTION ASCENDING

Reduce 1,000,201 oz. to tons, hundredweight, pounds, and ounces.

16) 4)1000201
4)250050 — 1
100)62512 lb. — 2 } 9 oz.
20)625 cwt. — 12 lb.
31 tons — 5 cwt.

1,000,201 oz. = 31 tons 5 cwt. 12 lb. 9 oz.

Explanation: In 1,000,201 oz. there are as many pounds as there are 16's in 1,000,201, that is, 62,512 lb. and 9 oz. over. In 62,512 lb. there are as many hundredweight as there are 100's in 62,512, that is, 625 cwt. and 12 lb. over. In 625 cwt. there are as many tons as there are 20's in 625, that is, 31 tons and 5 lb. over.

EXERCISE 28

1. Repeat the table of Avoirdupois weight.
2. Change 25 tons 59 lb. 16 oz. to ounces.
3. Reduce 12,425 lb. to tons, hundredweights, pounds.
4. How many pounds, etc., in 8465 drams?
5. In 5 tons 10 lb. how many stone (14 lb.)?
6. Reduce 19 cwt. 15 lb. 3 oz. to drams.
7. In 147 cwt. how many grains?
8. Change 55,464 grains to pounds. etc,
9. Reduce 346,578 oz. to tons, etc.

CAPACITY

There are two sets of measures of *capacity*, one for liquids and one for dry commodities.

DRY MEASURE

2 pints (pt.)	1 quart (qt.)
4 quarts	1 gallon (gal.)
2 gallons	1 peck (pk.)
4 pecks	1 bushel (bu.)

LIQUID MEASURE

2 pints	1 quart
4 quarts	1 gallon

NOTE: The standard measure of capacity is the Imperial gallon containing 10 pounds of distilled water. A cubic foot of water weighs almost 1000 oz., and contains almost 6¼ gallons.

The following table gives the equivalent in pounds of a bushel of the article mentioned, as fixed by law:

Wheat, Beans, Pease, Clover Seed, Potatoes, Beets, Carrots, Turnips	60 lb.
Rye, Indian Corn	56 lb.
Barley, Buckwheat, Timothy Seed	48 lb.
Onions . . . 50 lb. Oats	34 lb.

ORAL EXERCISE

1. How many pints make 1 qt. (dry measure)?

2. How many quarts make 1 pk.? How many pints make 1 pk.? How many pints make 1 bu.?

3. How many pecks in 3 bu.? How many pounds in 4 bu. of corn? How many pounds in 3 bu. of oats?

4. How many pints in 15 qt. of milk?

5. How many quarts make 2 barrels (bbl.) of $26\frac{1}{4}$ gal. each?

6. How many pounds in 6 bu. of wheat? 5 bu. of barley? 6 bu. of potatoes? 9 bu. of clover seed? 8 bu. of pease? 3 bu. of timothy seed?

EXERCISE 29

1. Change 527 pt. to bushels, pecks, quarts, and pints.

2. In 4657 qt. of oats how many bushels, etc.?

3. Reduce 16 bu. 7 gal. 3 qt. to quarts.

4. Three casks of wine hold respectively 40 gal., 35 gal., and 27 gal. How many bottles each holding 2 pints can be filled from all the casks?

5. What is the value of 8 gal. of coal oil at 3c. a pint?

6. Quart bottles are filled from a 54 gal. cask of wine. How many bottles will be filled and what are they worth at 50c. a bottle?

7. A farmer sold 840 lb. of wheat at $1·25 a bushel, 340 lb. oats at 60c. a bushel, 432 lb. barley at 75c. a bushel, and 1400 lb. rye at 65c. a bushel. He received in payment $20 in cash and sugar at $5 a 100 lb. How many pounds of sugar did he get?

8. If one quart of coal oil is worth 6c., what is a barrel worth?

9. Change 74806 pt. to bushels, etc.

10. Change 245 bu. of wheat to ounces.

11. If 3 fourths of a bushel of wheat is worth 90c., find the value of 1 pt., and also of 1 qt.

12. A horse is given 3 feeds of oats a day, each containing 1 gal.; how long will 27 bu. feed him?

13. What is the value of a load of oats weighing 1037 lb. at 12c. a peck? *Solve in two ways.*

14. A farmer's wife sold 15 lb. 12 oz. of butter at 32c. a lb., and got in exchange for it molasses at 63c. a gal. What was the price of the butter an oz.? How many gallons did she get?

15. Find the cost of 4 pk. 5 qt. 1 pt. of berries at 14c. a qt.

16. A farmer sold a load of rye weighing 4032 lb. when rye was worth 75c. a bu. In weighing the grain the buyer by mistake took it as barley and paid for it at 49c. a bu. How much did the farmer gain or lose by the error?

LONG OR LINEAR MEASURE

12 inches (in.)	1 foot (ft.)
3 feet	1 yard (yd.)
$5\frac{1}{2}$ yards	1 rod (rd.)
320 rods	1 statute mile (mi.)

NOTE: In the measurement of land 22 yards = 1 chain = 100 links.
In the measurement of horses 4 inches = 1 hand.

ORAL EXERCISE

1. How many feet in $5\frac{1}{2}$ yd.? in a rod?

2. How many inches in a yard?

3. How many inches in three quarters of a foot? two thirds of a foot? five sixths of 10 ft.? 3 ft.?

4. How many inches in 3 yd.? 6 yd.? 10 yd.?

5. How many feet in 5 yd.? 15 yd.? 25 yd.?

6. How many yards in 54 ft.? 75 ft.? 240 ft.?

7. How many miles in 640 rd.? 960 rd.? How many miles and rods in 800 rd.?

8. Make a rod measure of a cord, tying knots to indicate feet and yards. How many yards in a rod? How many feet in a rod?

9. With this cord measure (*a*) the width of the street or road; (*b*) the distance from the school door to the farthest corner of the school grounds.

10. Without using any measure draw on the black-board lines 1 ft. long, 1 yd. long, 3 yd. long, 1 rd. long. Then with the measure test your drawings. *There should be many tests in measuring of this kind.*

11. If 2 thirds of a yard of cloth cost 12c., what should 24 ft. cost? *Solve this in two ways.*

12. How many pieces of string 2 thirds of a yard long can be made out of a piece 8 yd. long?

13. A lot is 6 rd. wide and 10 rd. long. How often will a boy have to walk around it to walk 4 miles? 6 miles?

14. How many minutes will it take to walk 3 miles at the rate of 16 rd. a minute?

15. What is meant by saying that a lot is 8 ft. by 3 ft.? 9 rd. by 5 rd.? How many feet of wire-fencing will it take to reach around my garden which is 8 yd. by 4 yd.?

EXERCISE 30

1. How many feet in a mile? in 6 miles 980 ft.?

2. A field is a mile long and half a mile wide. How many rails each 12 ft. long will be required to reach around it? How many of such rails will it take to build a fence 5 rails high around the field?

3. How many times is $3\frac{1}{2}$ yd. contained in 21 yd.? How many half-yards are there in $3\frac{1}{2}$ yd.? How many half-yards in 21 yd.? How many times is 7 half-yards contained in 42 half-yards? What has been done to both divisor and dividend in obtaining this quotient? (See pages 50 and 70.)

4. Divide 70 by $3\frac{1}{2}$, 198 by $5\frac{1}{2}$, 792 by $16\frac{1}{2}$, 68 by $4\frac{1}{4}$, 89 by $30\frac{1}{4}$.

Example 1: Reduce 646 yd. to rods.

$5\frac{1}{2}$)646 yards
2 2
11)1292
117 rods — 5

What is the 1292? What then must the 5 be? How many rods and yards are there in 646 yd.? 117 rd. $2\frac{1}{2}$ yd. or 117 rd. 2 yd. 1 ft. 6 in.

Example 2: Change 6837 yd. to rods.

$5\frac{1}{2}$) 6837
2 2
11)13674 half-yards
1243 rods 1 half-yard or
1243 rods 0 yd. 1 ft. 6 in.

EXERCISE 31

1. Change 675 ft. to rods by one division.
2. Reduce 8765432 inches to miles, etc.
3. In 27 miles 4 yd. 2 ft. 6 in. how many inches?
4. What is the difference between 3 miles 319 rd. 5 yd. 1 ft. 6 in., and 4 miles? *Do this mentally.*

SURFACE, SQUARE OR LAND MEASURE

RECTANGLES

How many sides have these figures? How many corners (angles)?

What is a square inch? a square foot? a square mile? Make on your slate or exercise book a square inch. Make on the black-board a square foot and a square yard.

Draw a square foot. Divide each of its sides into inches. How many inches will there be in each side? Join the opposite points of section by lines from top to bottom and from side to side.

How many smaller rectangles have you made? What kind of rectangle is each of these smaller rectangles? How many are there? How many square inches in a square foot?

Now draw on the black-board or floor a square yard, divide each of its sides into feet, and join the opposite points of section. How many feet in each side? How many small rectangles have thus been made? Are they oblongs or squares? How many square feet in a square yard?

Draw, on a scale of half an inch to a yard, a figure $ABCD$ to represent a square rod. Starting at A, divide each of the sides AB and AD so as to show the number of yards in a rod. Starting from B, divide the side BC in a similar manner. Beginning at D, divide DC in the same way. Then by lines, a yard apart, drawn from top to bottom and from left to right, divide the whole of the figure into smaller figures. How many of these smaller rectangles are squares? How much space does each of these squares contain? How much space do all of these smaller squares contain? How much space does each of the remaining rectangles (omitting the one in the lower right-hand corner) contain? How many square inches do the ten contain? What is the length of the smallest square? What is its breadth? What part of a square

inch does it contain? How many square inches does the whole figure *ABCD* contain? How many square yards are there in a square rod?

SURFACES

The **area** of a surface is the number of **units of surface** which the surface contains.

The units of surface commonly used are the square yard, the square rod, the acre, and the square mile.

TABLE OF SURFACE, SQUARE OR LAND MEASURE

144	square inches	= 1 sq. ft.
9	square feet	= 1 sq. yd.
30¼	square yards	= 1 sq. rod
160	square rods	= 1 acre
640	acres	= 1 sq. mile

NOTE: In some parts of Canada a square mile of land is called a Section.

ORAL EXERCISE

1. How many square feet in 8 sq. yd.? in 12 sq. yd.? in 10 sq. yd.?

2. How many square yards in 4 sq. rd.? in 8 sq. rd.? in 28 sq. rd.?

3. How many square feet in 432 sq. in.? in 720 sq. in.? in 864 sq. in.?

4. In 81 sq. ft. how many square yards? In 54 sq. ft. how many square yards?

5. In 640 sq. rd. how many acres? in 1600 sq. rd.? in 6400 sq. rd.?

6. What part of a square mile is 160 acres?

7. In 1 sq. mile how many acres?

8. Change 6 acres to square rods.

EXERCISE 32

1. In 3456 sq. yd. how many square rods and square yards?

2. How many square yards in an acre?

3. In 8 sq. ft. 125 sq. in., how many square inches?

4. Reduce 37,894,635 sq. in. to acres, etc. *Have no fractions in the answer.*

5. Find the cost of 20 miles of telephone wire at 35c. a pound if one pound will stretch 80 ft.

6. Find the cost of a square mile of land at $10·25 an acre.

7. Reduce 5 ac. 39 sq. rd. 3 sq. yd. 7 sq. ft. 100 sq. in. to square inches. Prove the result by reducing the square inches to acres, square rods, etc.

8. In two quarter-sections of land, how many square rods?

9. Find the difference between 10 ac. 159 sq. rd. 30 sq. yd. 2 sq. ft. 36 sq. in. and 11 acres. *Do this mentally.*

SOLID OR CUBIC MEASURE

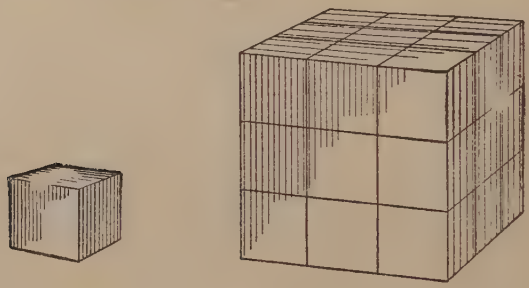
In the school-room there is a block for teaching square measure and cubic measure. What are its dimensions? How many faces has it? What is the area of each of these faces? How many solid corners or angles has the block? If each edge of the block were an inch long, what name would you give to the block? if each edge were a yard?

A cubic inch is the amount of space taken up by a cube each edge of which is an inch. What space is taken up by a cubic foot? a cubic yard?

How many cubic inches are there in a part of your block 1 in. wide, 12 in. long, and 1 in. thick? in a similar section or part, 2 in. thick? 3 in. thick? 4 in. thick?

How many in a section 12 in. long, 12 in. wide, and 1 in. thick?

How many cubic inches are there in a section of a cubic foot 12 in. long, 12 in. wide, and 2 in. thick? in a similar section 3 in. thick? 4 in. thick? 12 in. thick? How many cubic inches in a cubic foot?



These figures represent, respectively, a cubic foot and a cubic yard. How many cubic feet are there in a section of the cubic yard 3 ft. long, 3 ft. broad, and 1 ft. thick? in a section 3 ft. long, 3 ft. broad, and 2 ft. thick? How many cubic feet in a cubic yard?

Firewood, rough stone, and gravel are measured by the cord. When wood is cut for market the sticks are, as a rule, 4 ft. in length. A pile of such wood 4 ft. high and 8 ft. long contains a cord. Study this figure of a cord of wood carefully, and then answer these questions:

If a section 4 ft. high, 4 ft. wide, and 1 ft. thick be cut off the right-hand end of the pile, how many cubic feet are

in it? If this end section be 2 ft. thick, how many cubic feet are in it? if it be 8 ft. thick? How many cubic feet are there in a cord of wood or stone?

A section, 4 ft. by 4 ft. by 1 ft., of a pile such as is shown in the figure is said to contain 1 cord foot. How many cubic feet in a cord foot?

TABLE OF CUBIC OR SOLID MEASURE

1728 cubic inches (cu. in.) . .	1 cu. ft.
27 cubic feet	1 cu. yd.
128 cubic feet	1 cord

EXERCISE 33

1. Change 6 cords 106 cu. ft. to cubic feet.

2. Reduce 9856 cu. ft. to cords.

3. In 63,784 cu. in. how many cubic yards, etc.?

4. A pile of wood is 40 ft. long, 4 ft. high, and 4 ft. wide; how many cords does it contain? *Do this mentally.*

5. A pile of stone is 56 ft. long, 4 ft. high, and 4 ft. wide; how many cords are there in it? *Solve this mentally.*

6. If the pile were 8 ft. long, 8 ft. high, and 4 ft. wide, how many cords would there be in it? *Work this mentally.*

7. A teamster drew a cubic yard of gravel. Another teamster drew a quarter of a cord of gravel. In cubic ft., what was the difference between the loads?

8. A pile of wood one fourth of a mile long, 12 ft. high, and 16 ft. wide is to be shipped by rail. How many cars, each taking 10 cords, will be required to carry the wood? How many times the length of an ordinary cord is the pile? how many times the height? how many times the width?

9. How many cords in a pile 128 ft. long, 8 ft. high, and 4 ft. wide?

10. Measure a pile or piles of wood or stone, at home, in the school yard, or in some other place, and then find how many cords are in it or them. Show the measurements and the solutions to your teacher.

TIME MEASURE

60	seconds (sec.)	1 minute (min.)
60	minutes	1 hour (hr.)
24	hours	1 day (da.)
7	days	1 week (wk.)
12	months	1 year (yr.)
365	days	1 common year
366	days	1 leap-year
100	years	1 century

NOTE 1: The standard unit for measuring time is the *mean solar year*, which is equal to 365 days, 5 hr. 48 min. 46 sec., or nearly $365\frac{1}{4}$ days.

NOTE 2: The following lines are worth committing to memory:

Thirty days hath September,
April, June and November;
February has twenty-eight alone—
All the rest have thirty-one;
But leap-year coming once in four,
February then has one day more.

NOTE 3: Every year whose number is divisible by 4 is a leap-year, unless the number of the year ends in two ciphers (as 1600, 1900), in which case the date number must be divisible by 400.

NOTE 4: Time *before noon* is indicated by a.m.; *noon*, by m.; time *after noon*, by p.m.

ORAL EXERCISE

1. Express 1 wk. as minutes; 15 sec. as a part of a minute; 7200 sec. as hours.

2. How many months old are you this month?

3. How many weeks and days are there in 87 days?

4. How many school-days are there in 8 weeks?

5. How many minutes in 3 fourths of an hour? 2 thirds of an hour? 5 sixths of an hour?

6. How many months in 3 fourths of a year? 5 sixths of a year?

7. What part of a century is fifty years? 25 years? 75 years?

8. Make a list of the common and leap-years in the following: 1896, 1900, 1803, 1906, 2400 and 2764.

9. If September begins on Wednesday, what are the Monday dates? the Saturday dates?

10. What century are we living in? When did it begin? On what day will it end?

EXERCISE 34

1. How many days from 19th August to 25th December? from 15th November, 1903, to 4th March, 1904? from 28th September, 1896, to 23rd November, 1898? In finding the time between such dates, leave out either the first or last day. From the 2nd to the 25th is 23 days.

2. In 543,267 minutes how many years, days. hours, etc., taking $365\frac{1}{4}$ days for a year? *Have no fractions in your answer.*

3. Find the exact date from 4th January:

(*a*) 3 months back.

(*b*) 2 months and 12 days back.

(*c*) 3 years and 2 months back.

4. Reduce 4 yr. 264 da. 48 min. to minutes.

5. If 15 horses eat 11 bu. of oats in 9 days, how long will 44 bu. last 45 horses at the same rate? *Solve this mentally.*

6. If telegraph poles are 198 ft. apart, and a train passes one every 3 seconds, what is the train's rate an hour?

7. If 19 men can do a piece of work in 76 days of 7 hours each, how many men working at the same rate will it take to do the work in 133 hours? *Solve in two ways.*

8. A city newspaper office must get out an edition of 126,000 papers. It has two presses, each with a capacity of 300 papers a minute. How long (in hours) will it take to print the edition?

9. If 145 bu. of turnips last 53 oxen a fortnight, how long will 435 bu. last 371 oxen? *Solve this mentally.*

10. A train leaves a city at 5·15 p.m. and reaches another city 110 miles distant at 7·51 p.m. Allowing 6 minutes for stopping at stations, what is the rate of travel an hour?

11. TIME SHEET

Men	Mon.	Tues.	Wed.	Thu.	Fri.	Sat.	Wages 20c. an hour
John Smith . .	9	9½	8	10	9½	8	
James Grant .	8½	9	7½	9	8¾	7	
Robt. Jones . .	10	9½	8½	7½	10	9½	
Rich. Owens. .	7½	9	10	8	9	10	
Wm. Main . . .	6	8½	0	7½	8¾	0	
A. A. James. .	....	7½	10	8½	9	7	

From the foregoing weekly time sheet of a certain factory find:

(*a*) The total week's wages of the 6 men.

(*b*) How much money Wm. Main lost by being absent on Wednesday and Saturday, 10 hours making a full day.

(*c*) How much more did R. Owens earn than A. A. James? than Wm. Main?

(*d*) How many full days' work is equal to the whole time of the six men?

12. A plumber is engaged on a piece of work from 10 a.m. on Monday till 2 p.m. on Thursday of the following week. His work-day is from 8 a.m. to 12 m., and from 1 p.m. to 5 p.m., except on Saturday when it ends at 12 m. He charges 60c. an hour. How much will he receive?

ANGULAR MEASURE

This table is used in measuring angles, and in determining latitude, longitude, direction, the position of vessels at sea, etc.

60 seconds (″)	1 minute, or 1′
60 minutes (′)	1 degree, or 1°
90 degrees (°)	1 right angle

EXERCISE 35

1. How many degrees in 540′ ?

2. How many minutes in 750″ ?

3. With the protractor make an angle of 40°, of 50°, of 45°, of 120°.

4. Reduce 6° 16′ 15″ to seconds.

5. Measure, with the protractor, the angles made on the black-board by your teacher.

MISCELLANEOUS TABLE

12 single things	1 dozen (doz.)
12 dozen	1 gross (gro.)
12 gross	1 great gross
20 single things	1 score
24 sheets of paper	1 quire (qr.)
20 quires	1 ream (rm.)
196 lb. flour	1 barrel (bbl.)
200 lb. pork	1 barrel (bbl.)

EXERCISE 36

1. A stationer buys paper at 40c. a quire and sells it at 3c. a sheet. What is his gain a ream ?

2. At a High School examination, there were 200 candidates. On an average, each used 10 sheets for each subject. There were 15 subjects to be examined on. How many reams of paper were used ?

3. In 2940 pounds of flour how many barrels ?

4. Reduce 11,520 sheets to reams.

5. The pulse of a healthy person beats 70 times in a minute. At this rate, how many times will it beat in a leap-year? How many times will it beat in the four successive years beginning with 1904?

CANADIAN MONEY

10 mills	1 cent
100 cents	1 dollar

BRITISH (STERLING) MONEY

4 farthings	1 penny (d.)
12 pence	1 shilling (s.)
20 shillings	1 pound (£)

NOTE: £1=$4·86⅔.

ORAL EXERCISE

1. How many pence in 2 shillings? in 5 shillings? in 20 shillings? in £1?

2. How many shillings in 24 pence? in 60 pence?

3. How many pounds in 60 shillings? in 100 shillings? in 90 shillings?

4. How many pence in £2?

5. How many shillings in £3 10s.?

6. How many mills in 85 cents? in $1·20? in $2·36?

EXERCISE 37

1. Reduce to pence: £8, £5 4s., £12 5s. 3d.

2. How many shillings in 1680 farthings?

3. Reduce 9285 pence to £ s. d.?

4. Find the value in £ s. d. of 157 yards of ribbon at 6 pence a yard.

EXERCISE 38 (REVIEW)

NOTE: Before attempting to solve any problem read it very carefully. What seems to be difficult may really be easy.

1. How many yards of carpet 3 fourths of a yard wide are needed to carpet a room 24 ft. by 18 ft., if the carpet runs lengthwise? *Do this mentally.*

2. Find the cost of carpeting a room 20 ft. by 18 ft. with carpet 27 in. wide at 90c. a yard, the strips being laid crosswise.

3. A train travels at the rate of $52\frac{1}{2}$ miles in $1\frac{3}{4}$ hours. What is its rate a minute? How many hours and minutes will it take to travel 208 miles? how many minutes?

4. If 1 lb. 8 oz. of cheese cost 24c., find the cost of 4 lb.

5. Find the cost of a car load of coal weighing 35,000 lb. at $5.50 a ton.

6. A man having a salary of $1500 a year spends 1 fourth of it for board and lodging, 1 tenth for clothing, and 3 twentieths for other things. How much money does he spend?

7. What part of 1000 is 250? What part is 750? Find the cost of 18,750 ft. of lumber at $30 a 1000 ft.

8. A man borrows money on 1st April and agrees to pay it in 90 days. On what day should he pay it?

9. What will it cost to fence a lot 42 ft. by 120 ft. at 85c. a yd.? *Do this mentally. Solve it afterwards using pencil.*

10. How many cords of wood in a pile 56 ft. long, 12 ft. wide, and 8 ft. high?

11. When coal sells for $7 a ton, what is the cost of 200 lb.? *Solve mentally.*

12. A, B, C go into business as partners. They put into the business $4000, $4500, $6500 respectively. At the end of two years they divide a profit of $6000. Find each person's share of this profit.

13. Mr. Jones borrows $1200 from Mr. Richardson, agreeing to pay him $8 a year for the use of every $100 he borrows. At the end of 5 years how much will he have to pay to wipe out the whole debt?

14. By buying eggs at 24c. a doz. and selling them at 60c. a score, a dealer makes a profit of $10.01. How many eggs does he sell? *Solve this mentally.*

15. If the quotient is 9756 and the dividend 741,456, what is the divisor?

16. In 3,947,654 in., how many miles, etc.?

17. In 3,946,834 cu. in., how many cubic yards? *In dividing, use factors.*

18. A grocer bought potatoes at 40c. a bushel and sold them at 15c. a peck. How much did he gain on 425 bushels?

19. Find the cost of the beans required to plant 5 acres, allowing 12 quarts to an acre, the beans being worth 45c. a peck.

20. If a tree sparrow eats in a day seeds which would produce 75 weeds, a dozen such birds will in July destroy seeds that would produce how many weeds?

21. If a woodpecker eats on an average 1690 insect pests in a day, how many will 250 woodpeckers eat during May?

22. Find the total cost of 5800 lb. of hay at $10 a ton, 625 lb. of pork at $12 a cwt., and 5500 ft. of lumber at $16 per M.

23. The assessed value of a property is $2500. On every dollar of assessment 15 mills of tax is levied. What are the taxes on the property?

24. In the shortest time possible find the results:

(1) $ 123596·78
91327·23
234·56
78·90
876543·29
50022·33

Sum, $

(2) 2469×6=
9839×7=
20135×5=
125839×9=
257×8=

Total,

(3) 24036890741
9302853729

Difference,

(4) 873257÷927

25. Hugh McKinnon bought from James O'Reilly the following:

305 bu. 15 lb. wheat at 52c.
17 bu. 17 lb. oats at 44c.
10 bu. 6 lb. barley at 48c.
7 bu. 14 lb. Indian corn at 96c.
320 bu. 15 lb. potatoes at 64c.

Find the total amount.

26. At \$1 a 100 lb. what will 1 lb. of pork cost in cents? at \$2 a 100 lb.? at \$4? at \$6? at \$10?

27. If you know the price, in dollars, of 100 lb., what is the price in cents of 1 lb.?

COMPOUND DENOMINATE NUMBERS

What is a denominate number? What is a compound denominate number? (See page 63.)

You have already noticed that in ordinary numbers 10 units of one kind make 1 unit of the next higher kind. Is that true in the case of compound denominate numbers? In the number 13 mi. 256 rd. 2 ft. 6 in., what name is given to the lowest unit? to the next higher? to the next? to the highest?

How many units of each kind make one of the next higher?

Addition, subtraction, multiplication, or division of compound denominate numbers is usually called **compound addition, compound subtraction, compound multiplication,** or **compound division.**

In simple addition, simple subtraction, simple multiplication, and simple division, numbers are reduced to other denominations by multiplying or dividing by 10; in *compound addition, compound subtraction, compound multiplication,* and *compound division,* numbers are

reduced to higher or lower denominations by multiplying or dividing by 12, $5\frac{1}{2}$, 320, 144, 9, $30\frac{1}{4}$, etc., as may be needed.

EXERCISE 39

1. Add:

bu.	pk.	qt.	pt.
3	5	6	1
8	4	1	0
7	3	5	1
9	4	3	1

2. Add:

tons	cwt.	lb.
16	17	74
13	10	20
17	15	19
84	0	8
11	11	36

3. Subtract:

yd.	ft.	in.
15	1	5
13	2	7

4. Add:

mi.	rd.	yd.	ft.	in.
2	27	1	2	8
1	146	2	1	6
8	90	4	0	4
7	152	1	2	9

5. Add:

£	s.	d.
19	5	11
27	14	6
8	0	$7\frac{1}{2}$
49	18	$6\frac{3}{4}$

NOTE: Part of the sum in Question 4 will be $2\frac{1}{2}$ yd. Change the $\frac{1}{2}$ yd. to 1 ft. 6 in. and add this to the rest of the sum, making each of its parts a whole number. In the Compound Rules there should be no fractions in the answers, except with the lowest unit.

6. Subtract:

cu. yd.	cu. ft.	cu. in.
37	19	859
34	25	1381

7. Subtract:

£	s.	d.
276	3	$4\frac{1}{4}$
197	14	$9\frac{1}{2}$

8. Multiply:

mi.	rd.	yd.	ft.	in.
21	187	3	1	9
				9

9. Divide:

tons	cwt.	lb.	oz.
12)37	16	87	12

10. Subtract 4 yr. 60 da. 2 hr. 54 min. from 6 yr. 64 da. 20 hr. 50 min.

11. Multiply 36 deg. 54 min. 21 sec. by 11.

12. Multiply 30 sq. yd. 5 sq. ft. 129 sq. in. by 9.

13. A merchant sold to one lady 27 yd. 3 in., and to another 3 yd. 1 in., of a piece of cloth of 100 yd. in length. By how much did the part left exceed the part sold?

14. Multiply 9 ac. 144 sq. rd. 27 sq. yd. 8 sq. ft. 127 sq. in. by 240, using factors.

15. Multiply 1 mi. 100 yd. 2 ft. by 1000, using factors.

16. From a pile of wood containing 36 cords 4 cord ft. there was sold 10 cords 6 cord ft. 12 cu. ft. How much remained?

17. What is the difference between 7 times 5 sq. mi. 250 sq. ac. 145 sq. rd., and 5 times 456 ac. 137 sq. rd. 29 sq. yd.?

18. Multiply 11 sq. yd. 3 sq. ft. 8 sq. in. by 365.

Process:

sq. yd.	sq. ft.	sq. in.	
11	3	8	
		10	
113	3	80	= 10 times 11 sq. yd. 3 sq. ft. 8 sq. in.
		10	
1133	8	80	= 100 " " " " "
		3	
3401	7	96	= 300 " " " " "
680	3	48	= 60 " " " " "
56	6	40	= 5 " " " " "
4138	8	40	= 365 " " " " "

NOTE: In what other ways could 60 times the multiplicand be found? also, five times the multiplicand?

19. Multiply 1 mi. 150 yd. by 503.

20. Multiply 3 tons 1 cwt. 14 lb. by 2347.

21. Divide 371 ft. 6 in. by 9.

22. Divide 94 mi. 1400 yd. 2 ft. 5 in. by 7.

23. Divide 189 tons 1678 lb. by 387. *Show all the work.*

24. How many times is the length 3 ft. 7 in. contained in 21 ft. 9 in.? First express both lengths in inches.

25. Divide 2 days 7 hr. 9 min. 18 sec. by 3 hr. 27 min. 42 sec.

26. How many parcels, each weighing 41 lb. 8 oz., can be made up of goods weighing 1 ton, and what weight will remain?

27. Divide 110 mi. 252 rd. 4 yd. by 2 mi. 42 rd. 5 yd.

28. How many times is 16 hr. 30 min. 20 sec. contained in 37 da. 19 hr. 48 min. 20 sec.?

29. Divide 76 yd. 2 ft. 9 in. of cloth between two persons, giving one twice as much as the other.

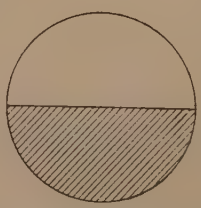
30. If a wheel is 8 ft. 9 in. in circumference, how many revolutions will it make in going a mile?

31. On Monday a hotel used 4 gal. 2 qt. of milk; on Tuesday 5 gal.; on Wednesday 5 gal. 1 qt. 1 pt.; on Thursday 6 gal. 3 qt.; on Friday 7 gal. 5 pt.; on Saturday 3 gal. 2 qt. 1 pt.; and on Sunday 2 gal. 1 pt. What was the milk bill for the week at 14 qt. tickets for one dollar?

32. If the large wheels of an automobile are 7 ft. in circumference, how many times will each one turn in going 28 miles?

33. How many years, months, and hours from 3 o'clock p.m. on 15th June, 1852, to 10 o'clock a.m. of 22nd March, 1860?

FRACTIONS



Out of paper make several squares, circles, oblongs, or long strips.

ORAL EXERCISE

1. Fold a square once by placing the lower edge carefully in line with the upper. Into how many parts has the square been divided? Which of these parts is the larger?

2. Can the square be folded in any other way so as to divide it into parts of the same size as those which you got by your first folding? Show that each of these is the same size as each of the first.

3. What name is given to each of these parts?

4. Divide a circle, an oblong, a strip, etc., into halves.

5. Will one half of the circle be the same size as the half of the square? What halves are equal to one another?

6. How would you find the half of any quantity?

7. What is one half of \$4? of 8 yd.? of 12 lb.? of 5 qt.? of 16? of 9?

8. If one half a pound of beef costs 6c., what is the price of one pound? of 3 pounds? of 2 and one half pounds?

ORAL EXERCISE

1. Fold a square first by putting the lower edge carefully in line with the upper, then by putting the left edge carefully in line with the right. Into how many parts is the square now divided? Which of these parts is the largest?

2. In what other ways can the square be folded to get parts equal to the parts you now have? How can it be shown that each of these parts is equal to each of the first?

3. What is the name given to each of these parts?

4. Divide a circle, an oblong, a strip, etc., into quarters or fourths.

5. Cut off one fourth of the strip, three fourths of the circle, and two fourths of the oblong.

6. How many fourths are there in one half? in two halves?

7. One fourth is what part of one half? How many halves of one half are there in three fourths?

8. If to one half you add one fourth, how many fourths do you get? If from one half you take away one fourth, what remains? If from the whole you take away one fourth, what remains?

9. How many times is one fourth contained in three fourths? In 1?

ORAL EXERCISE

1. How many halves (of one) are there in a whole or one? in two wholes or two? in three? in 6? in 10? in one and one half? in 3 and one half? in 5 and one half?

2. If from one and one half you take away two halves, what remains?

3. If to 7 and one half you add two and one half, what is the result?

4. How many fourths (of one) are there in one whole or 1? in 2? in 3? in 7? in 4 and one fourth? in 3 and two fourths? in 5 and three fourths? in one and one half? in 2 and one half?

5. What is the result when three fourths are taken from 2? when one half is taken from 3 and one fourth? when 2 and three fourths are added to six and one half?

6. Out of 4 halves how many wholes or ones can you get? out of 6 halves? out of ten halves? out of 16 halves? out of 5 halves? out of 17 halves?

7. Out of 8 fourths how many ones can you get? out of 12 fourths? out of 24 fourths? out of 7 fourths? out of 18 fourths? out of 19 fourths?

8. At one fourth of one dollar each, how many books can I buy for 2 dollars?

9. If one fourth of a man's hay crop was sold for $70, what was the whole crop worth at the same rate?

10. If three fourths of a yard of cloth costs 30c., what is the cloth worth a yard?

ORAL EXERCISE

1. Divide a square vertically into three equal parts. What is the name given to each of these parts ?

2. How many thirds are there in 1 ? in 2 ? in 4 ? in 7 ? in 2 and one third ? in 6 and two thirds ?

3. How many wholes or ones can you get out of 6 thirds ? out of 12 thirds ? out of 15 thirds ? out of 14 thirds ? out of 19 thirds ?

4. Divide a strip of paper into thirds ? How do you get one third of any quantity ? What is one third of $9 ? of 12 acres ? of 18 bushels ? of 24 ? of 60 ?

5. If one third of a field can be ploughed in 3 days, how long will it take to plough 2 fields each of the same size as the first ?

6. If one third of a man's money is $6, how much money has he ?

7. A boy spent one third of his money and had 20 cents left. How much money had he at first ?

8. Of what number is 5 the one third ? 4 the two thirds ?

ORAL EXERCISE

1. Divide a square vertically into thirds and then fold it once horizontally by placing the lower edge in line with the upper. Into how many parts is the square now divided ? Are the parts all equal ? What name is given to each of these parts ?

2. How many sixths are there in one whole ? in one half ? in two halves ? in one third ? in two thirds ? in three thirds ?

3. One sixth is what part of one half ? of one third ? of two thirds ?

4. What remains when you take one sixth from one half ? one sixth from one third ? one half from two thirds ?

5. What is the result when you add one sixth to one third ? one sixth to one half ? one third to one half ?

6. What other names can be given to two sixths? to three sixths? to four sixths?

7. How many sixths are there in 2? in 2 and one sixth? in 3 and one half?

8. How many wholes or ones can we get out of 18 sixths? out of 30 sixths? out of 15 sixths?

9. What is the result when five sixths is taken from 2 and one sixth? when 2 and four sixths is added to 3 and one third?

10. Divide an oblong into sixths. What is the one sixth of 12? of 36? of 42?

ORAL EXERCISE

1. Divide a square into eighths? Cut off two eighths. What names can be given to the part cut off? to the part remaining?

2. How many eighths are there in one half? in two fourths? in three fourths?

3. One eighth is what part of one half? of three fourths?

4. What is one eighth of 16? of 48? of 96?

5. How many eighths are there in 3? in 5? in 2 and three eighths? in 1 and one half?

6. How many wholes can you get out of 16 eighths? out of 32 eighths? out of 28 eighths?

7. If three fourths of a yard of cloth costs 90c., what will five eighths of a yard be worth?

8. Draw an oblong whose width will be one half of its length. Draw also one whose width will be two thirds of its length.

9. Draw two lines the length of one of which will be three fourths that of the other.

10. Draw a line 5 inches long. Divide it first into fifths and then into tenths.

Halves, thirds, fourths, sixths, etc., are called **fractions.**

A fraction is one or more of the equal parts of a unit.

A fraction may also be considered as a *denominate number,* the unit for which is one of the equal parts of a larger unit. Just as we call three quarts a denominate number for which the unit is one quart, so three fourths is a denominate number for which the unit is one fourth.

It will be seen later that a fraction may be considered as the quotient obtained by dividing one number by another.

Fractions may be written in several ways. For instance, instead of writing "seven eighths" we may write 7 eighths or $\frac{7}{8}$, both of which are read in exactly the same way.

The form $\frac{7}{8}$ is the one generally used. Here are two numbers placed one below, and the other above a horizontal line. The two numbers are called the **terms** of the fraction.

The term *below* the line is called the **denominator,** that *above,* the **numerator** of the fraction.

What is the use of the number below the line? of the number above the line?

The names halves, thirds, fourths, tenths, etc. give the **denominations** of the fractions.

EXERCISE 40

1. Read the following: $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{9}$, $\frac{7}{11}$, $\frac{8}{15}$, $\frac{6}{21}$, $\frac{3}{50}$, $\frac{44}{100}$, $\frac{6}{5}$, $\frac{12}{10}$, $\frac{25}{20}$.

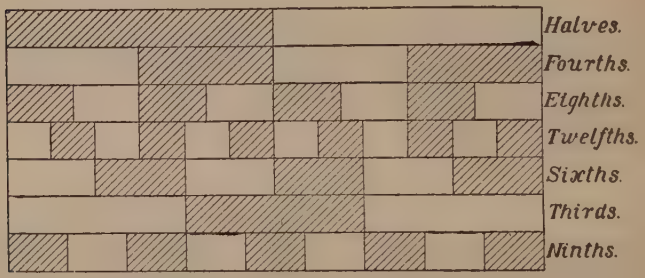
2. Write the following in figures: five sixths, nine twelfths, three eighths, seven twentieths, eighteen twenty-ninths, fifteen thirty-firsts, sixty-two seventy-fifths, eighty-three thirty-seconds, eleven fifty-firsts, forty-seven fifty-seconds, seventeen forty-thirds, twenty-three elevenths, fourteen halves.

3. Read the following and draw figures or fold paper to show what each means: $\frac{5}{16}$, $\frac{9}{8}$, $2\frac{1}{2}$, $3\frac{2}{3}$.

4. Write the following in figures: three, and one seventh; one, and two fifths; eleven, and one half; four, and three tenths; twenty-one twentieths; eight, and five thirty-seconds.

5. Take the following figures for denominators and supply numerators and read your fractions: 8, 7, 10, 16, 24, 32, 41, 61, 72, 83.

6. Take the following figures for numerators and supply denominators and read your fractions: 2, 5, 8, 13, 19, 22, 37, 50, 65, 90.



In the exercise with folded paper you found that $\frac{1}{2} = \frac{2}{4}$ or $\frac{3}{6}$ or $\frac{4}{8}$.

You can also show by using folded paper or by drawing figures like the above that $\frac{1}{2} = \frac{5}{10}$ or $\frac{6}{12}$ or what other fractions?

You also found that $\frac{1}{3} = \frac{2}{6}$, and, by folding paper or drawing figures like the above, you can show that $\frac{1}{3} = \frac{2}{6}$ or $\frac{3}{9}$ or $\frac{4}{12}$ or what other fractions?

So, also, $\frac{2}{3} = \frac{4}{6}$ or $\frac{6}{9}$ or $\frac{8}{12}$ or what other fractions?

So, too, $\frac{3}{4} = \frac{6}{8}$ or $\frac{9}{12}$ or what other fractions?

Now, show to what other fractions $\frac{3}{5}$, $\frac{5}{6}$, $\frac{3}{8}$ and $\frac{4}{9}$ are equal.

When one fraction is equal to another the two are said to be **equivalent,** that is, of equal value. From what you have already learned, state how a fraction may be changed to an equivalent one.

If $\frac{1}{2} = \frac{3}{6}$ or $\frac{5}{10}$, etc., then $\frac{3}{6}$ or $\frac{5}{10}$ must each equal $\frac{1}{2}$; and if $\frac{2}{3} = \frac{6}{9}$ or $\frac{8}{12}$, etc., then $\frac{6}{9}$ or $\frac{8}{12}$ must each equal $\frac{2}{3}$; and if $\frac{3}{4} = \frac{6}{8}$ or $\frac{9}{12}$, etc., then $\frac{6}{8}$ or $\frac{9}{12}$ must each equal $\frac{3}{4}$.

You can now state another method of changing a fraction to an equivalent one.

What principle in division is this like? (See pages 50, 51.)

When the numerator and denominator of one fraction are greater than those of its equivalent, the first fraction is said to be in **higher terms** than the second, and the second is said to be in **lower terms** than the first.

When two fractions have the same *denominator* they are said to be of the same *denomination.*

EXERCISE 41

1. How many eighths are there in $\frac{1}{2}$? in $\frac{3}{4}$? in $\frac{6}{16}$? in $\frac{12}{32}$?

2. How many twentieths are there in $\frac{1}{2}$? in $\frac{3}{4}$? in $\frac{2}{5}$? in $\frac{9}{10}$? in $\frac{16}{40}$?

3. Change, or reduce, each of the following fractions to twelfths: $\frac{2}{3}$, $\frac{1}{4}$, $\frac{5}{6}$.

4. Reduce each of the following to ninetieths: $\frac{3}{6}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{7}{10}$, $\frac{4}{15}$, $\frac{7}{30}$, $\frac{11}{45}$.

5. Reduce each of the following to hundredths: $\frac{3}{4}$, $\frac{4}{5}$, $\frac{9}{10}$, $\frac{8}{20}$, $\frac{8}{25}$.

6. Supply numerators in the following: $\frac{3}{4} = \frac{\quad}{16}$, $\frac{7}{8} = \frac{\quad}{24}$, $\frac{10}{11} = \frac{\quad}{88}$, $\frac{\quad}{12} = \frac{8}{6}$, $\frac{13}{40} = \frac{\quad}{10}$, $\frac{16}{56} = \frac{\quad}{7}$.

7. Supply denominators in the following: $\frac{5}{7} = \frac{\quad}{30}$, $\frac{3}{10} = \frac{\quad}{15}$, $\frac{4}{9} = \frac{\quad}{28}$, $\frac{12}{21} = \frac{\quad}{4}$, $\frac{18}{27} = \frac{\quad}{2}$.

8. Write, for each of the following, four equivalent fractions with higher terms: $\frac{2}{5}$, $\frac{1}{6}$, $\frac{5}{7}$, $\frac{3}{8}$.

9. Write, for each of the following, an equivalent fraction with lower terms: $\frac{5}{20}$, $\frac{10}{12}$, $\frac{8}{16}$, $\frac{27}{30}$, $\frac{20}{25}$.

10. Express, in the same denomination, each of the following pairs of fractions: $\frac{1}{2}$ and $\frac{3}{4}$, $\frac{1}{4}$ and $\frac{5}{8}$, $\frac{1}{2}$ and $\frac{2}{5}$, $\frac{3}{7}$ and $\frac{2}{5}$, $\frac{3}{4}$ and $\frac{5}{6}$, $\frac{2}{3}$ and $\frac{3}{4}$.

11. Examine the following table and extend it:

	4ths.	6ths.	8ths.	9ths.	10ths.	12ths.	14ths.	15ths.	16ths.	18ths.	20ths.	21sts.	22nds.	24ths.	25ths.
$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$		$\frac{5}{10}$	$\frac{6}{12}$	$\frac{7}{14}$		$\frac{8}{16}$	$\frac{9}{18}$	$\frac{10}{20}$		$\frac{11}{22}$	$\frac{12}{24}$	
$\frac{1}{3}$		$\frac{2}{6}$		$\frac{3}{9}$		$\frac{4}{12}$		$\frac{5}{15}$		$\frac{6}{18}$		$\frac{7}{21}$		$\frac{8}{24}$	
$\frac{1}{4}$	$\frac{1}{4}$		$\frac{2}{8}$			$\frac{3}{12}$			$\frac{4}{16}$		$\frac{5}{20}$			$\frac{6}{24}$	
$\frac{1}{5}$					$\frac{2}{10}$			$\frac{3}{15}$			$\frac{4}{20}$				$\frac{5}{25}$
$\frac{1}{6}$		$\frac{1}{6}$				$\frac{2}{12}$				$\frac{3}{18}$				$\frac{4}{24}$	
$\frac{1}{7}$							$\frac{2}{14}$					$\frac{3}{21}$			
$\frac{1}{8}$			$\frac{1}{8}$						$\frac{2}{16}$					$\frac{3}{24}$	
$\frac{1}{9}$				$\frac{1}{9}$						$\frac{2}{18}$					
$\frac{1}{10}$					$\frac{1}{10}$						$\frac{2}{20}$				
$\frac{1}{11}$													$\frac{2}{22}$		
$\frac{1}{12}$														$\frac{2}{24}$	

12. Use the above table to find equivalent fractions for $\frac{2}{3}$, $\frac{3}{3}$, $\frac{4}{3}$, etc.; for $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$, $\frac{5}{4}$, etc.; $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{5}{5}$, $\frac{6}{5}$, etc.; $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, etc., and so on.

NOTE: Observe that reducing fractions from one denomination to another is the same as the Reduction Ascending or the Reduction Descending of Denominate Numbers.

In a former exercise it was found that there were 6 halves in 3, 7 halves in 3 and one half, 28 fourths in 7, 23 fourths in 5 and three fourths, 12 thirds in 4, 20 thirds in 6 and two thirds, etc. These facts may now be expressed as follows: $3 = \frac{6}{2}$, $3\frac{1}{2} = \frac{7}{2}$, $7 = \frac{28}{4}$, $5\frac{3}{4} = \frac{23}{4}$, $\frac{12}{3} = 4$, $\frac{20}{3} = 6\frac{2}{3}$, etc.

Each of the quantities $3\frac{1}{2}$, $5\frac{3}{4}$, $6\frac{2}{3}$ is called a **mixed number,** being made up of a whole number and a fraction.

A fraction whose numerator is less than its denominator is called a **proper fraction.**

A fraction whose numerator is not less than its denominator is called an **improper fraction.**

Whole or *mixed* numbers may, therefore, be changed into what kind of fractions? How?

ORAL EXERCISE

1. Name several mixed numbers. Name also some improper fractions.

2. Read the following: $4\frac{1}{5}$, $3\frac{2}{3}$, $6\frac{4}{5}$, $2\frac{3}{4}$, $5\frac{5}{6}$, $4\frac{2}{7}$, $\frac{12}{3}$, $\frac{14}{4}$, $\frac{5}{5}$, $\frac{8}{1}$, $\frac{25}{8}$, $\frac{17}{7}$, $\frac{10}{10}$.

3. To what improper fractions are the mixed numbers in Question 2 equal?

4. To what whole or mixed numbers are the improper fractions in Question 2 equal?

5. Which of the following are proper fractions: $\frac{4}{2}$, $\frac{3}{7}$, $\frac{8}{6}$, 2, $\frac{18}{6}$, $4\frac{1}{3}$, $\frac{12}{10}$, $\frac{19}{6}$, $\frac{13}{4}$, $\frac{8}{8}$, 5?

6. To what mixed numbers are the improper fractions in Question 5 equal?

7. Express each of the following as an improper fraction: $2\frac{1}{2}$, $3\frac{2}{5}$, 4, $1\frac{4}{4}$, $2\frac{3}{11}$, $8\frac{2}{7}$, $9\frac{1}{11}$, 7, $3\frac{1}{4}$, 5.

8. Express each of the following fractions as a mixed number: $\frac{7}{3}$, $\frac{11}{4}$, $\frac{4}{4}$, $\frac{9}{5}$, $\frac{3}{1}$, $\frac{23}{6}$, $\frac{16}{8}$, $\frac{7}{1}$, $\frac{11}{11}$, $\frac{100}{20}$.

9. For each of the following, give an equivalent fraction with higher terms: $\frac{7}{4}$, $\frac{4}{3}$, $\frac{8}{5}$, $\frac{6}{6}$, $\frac{2}{1}$, $\frac{9}{7}$, $\frac{10}{1}$.

10. For each of the following, give an equivalent fraction with lower terms: $\frac{14}{4}$, $\frac{9}{6}$, $\frac{25}{10}$, $\frac{12}{3}$, $\frac{8}{8}$, $\frac{24}{18}$, $\frac{20}{8}$.

11. Express the fractions in each of the following pairs in the same denomination: $\frac{3}{2}$ and $\frac{6}{5}$, $\frac{2}{3}$ and $\frac{9}{6}$, $\frac{9}{4}$ and $\frac{7}{6}$, $2\frac{1}{3}$ and $1\frac{1}{2}$, $3\frac{1}{5}$ and $\frac{7}{10}$.

12. Express the fractions in each of the following groups in the same denomination: $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$; $\frac{2}{5}$, $\frac{3}{4}$, $\frac{3}{2}$. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{3}{8}$.

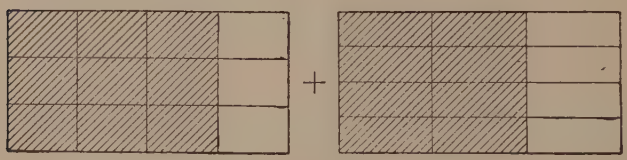
ADDITION AND SUBTRACTION OF FRACTIONS

In the addition and subtraction of whole numbers, only *like* numbers, that is those that have the same denomination or name, can be added together or subtracted from one another. (See page 7.) So also fractions can be added or subtracted only when they have the *same name* or denomination.

For instance, the sum of 3 fourths and 2 fourths is 5 fourths, but the sum of 3 fourths and 2 thirds is neither thirds nor fourths. If, however, in the latter case, both be changed to the same denomination (twelfths), the sum of 9 twelfths and 8 twelfths is 17 twelfths.

That is, $\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}$.

This can be shown as follows:



$\frac{3}{4}$ + $\frac{2}{3}$

$\frac{9}{12}$ + $\frac{8}{12}$

$\frac{17}{12}$ = $1\frac{5}{12}$

Again, $\frac{5}{6} - \frac{3}{8} = \frac{20}{24} - \frac{9}{24} = \frac{11}{24}$.

Show this by means of a figure.

What then must we do with fractions which are in different denominations before we can add or subtract them?

EXERCISE 42

NOTE: In answers it is usual to express all improper fractions as mixed numbers, and all fractions in their lowest terms.

Find the value of:

1. $\frac{1}{2} + \frac{1}{2}$.
2. $\frac{3}{8} + \frac{3}{8}$.
3. $\frac{3}{4} + \frac{1}{4}$.
4. $\frac{7}{8} - \frac{5}{8}$.
5. $\frac{8}{9} - \frac{4}{9}$.
6. $\frac{5}{10} - \frac{3}{10}$.
7. $\frac{5}{12} + \frac{3}{12}$.
8. $\frac{8}{12} + \frac{11}{12}$.
9. $\frac{5}{6} + \frac{7}{6}$.
10. $\frac{7}{10} + \frac{6}{10} + \frac{3}{10}$.
11. $\frac{9}{12} + \frac{8}{12} - \frac{7}{12}$.
12. $\frac{7}{8} - \frac{3}{8} + \frac{5}{8}$.
13. $\frac{1}{2} + \frac{1}{4}$.
14. $\frac{3}{4} - \frac{1}{2}$.
15. $\frac{3}{4} + \frac{5}{16}$.
16. $\frac{13}{16} - \frac{1}{2}$.
17. $\frac{5}{8} - \frac{7}{32}$.
18. $\frac{2}{3} + \frac{5}{6}$.
19. $\frac{2}{3} - \frac{4}{9}$.
20. $\frac{3}{2} + \frac{5}{8}$.
21. $\frac{4}{3} + \frac{3}{16}$.
22. $\frac{4}{5} - \frac{3}{20}$.
23. $\frac{5}{16} - \frac{3}{32}$.
24. $\frac{7}{8} + \frac{3}{4}$.
25. $\frac{3}{4} + \frac{5}{6} + \frac{7}{12}$.
26. $\frac{4}{9} + \frac{5}{18} - \frac{1}{6}$.
27. $\frac{3}{4} - \frac{1}{8} - \frac{3}{16}$.
28. $\frac{1}{2} + \frac{1}{3}$.
29. $\frac{1}{3} - \frac{1}{6}$.
30. $\frac{5}{4} + \frac{2}{5}$.
31. $\frac{3}{2} + \frac{4}{5}$.
32. $\frac{3}{4} - \frac{2}{5}$.
33. $\frac{3}{7} + \frac{5}{10}$.
34. $\frac{3}{8} + \frac{5}{6}$.
35. $\frac{5}{9} + \frac{4}{6}$.
36. $\frac{5}{6} - \frac{3}{4}$.
37. $\frac{3}{8} + \frac{7}{10}$.
38. $\frac{5}{8} - \frac{7}{12}$.
39. $\frac{5}{6} + \frac{3}{10}$.
40. $\frac{5}{6} + \frac{4}{9} - \frac{2}{3}$.
41. $\frac{2}{5} + \frac{3}{4} - \frac{1}{2}$.
42. $\frac{5}{8} + \frac{4}{5} - \frac{1}{4}$.

43. A grocer sold $\frac{5}{8}$ of a barrel of sugar to one customer and $\frac{3}{4}$ of a barrel to another. How many barrels did he sell altogether?

44. A farmer gave to one of his sons $\frac{1}{6}$, to another $\frac{1}{3}$, and to a third, $\frac{3}{10}$ of his farm. How much of the farm did he give away? What part had he left?

45. Between Ned's house and the school there is $\frac{1}{2}$ of a mile of board walk, $\frac{1}{4}$ of a mile of cement walk, and $\frac{7}{16}$ of a mile of gravel walk. What is the total distance?

46. A boy who had $\$\frac{4}{5}$ paid $\$\frac{3}{4}$ for a book. What part of a dollar had he left? How much money had he left?

47. In a plot containing $\frac{7}{8}$ of an acre, a man plants $\frac{5}{12}$ of an acre of potatoes and sows the remainder with turnips. How much ground has he under turnips?

ADDING AND SUBTRACTING MIXED NUMBERS

Example 1: Find the value of $6\frac{2}{3} + 3\frac{1}{2}$.

$$6\tfrac{2}{3} + 3\tfrac{1}{2}$$
$$= 6\tfrac{4}{6} + 3\tfrac{3}{6}$$
$$= 9\tfrac{7}{6} = 9 + 1\tfrac{1}{6} = 10\tfrac{1}{6}.$$

Example 2: Find the value of $4\frac{1}{5} - 2\frac{3}{4}$.

$$4\tfrac{1}{5} - 2\tfrac{3}{4}$$
$$= 4\tfrac{4}{20} - 2\tfrac{15}{20}.$$

Now, we cannot subtract $\frac{15}{20}$ from $\frac{4}{20}$, so we break 4 into $3 + 1$, and $1 = \frac{20}{20}$; then $4 = 3\frac{20}{20}$, and $4\frac{4}{20} = 3\frac{24}{20}$; thus we have

$$4\tfrac{4}{20} - 2\tfrac{15}{20}$$
$$= 3\tfrac{24}{20} - 2\tfrac{15}{20} = 1\tfrac{9}{20}.$$

The work may also be set down thus:

$$4\tfrac{1}{5} = 4\tfrac{4}{20}$$
$$2\tfrac{3}{4} = 2\tfrac{15}{20}$$
$$1\tfrac{9}{20}.$$

EXERCISE 43

Find the value of:

1. $12 + \frac{5}{6}$.	6. $5\frac{1}{3} - 3$.	11. $5\frac{3}{4} + 2\frac{1}{8}$.
2. $3 - \frac{4}{5}$.	7. $5\frac{2}{3} + \frac{5}{6}$.	12. $7\frac{3}{8} - 2\frac{19}{12}$.
3. $4 + 1\frac{3}{5}$.	8. $4\frac{5}{8} - \frac{1}{2}$.	13. $11\frac{3}{8} + 3\frac{5}{8}$.
4. $6 - 2\frac{2}{5}$.	9. $3\frac{1}{3} + \frac{7}{9}$.	14. $9\frac{2}{7} - 2\frac{7}{10}$.
5. $3\frac{1}{6} + 2$.	10. $3\frac{4}{5} - \frac{9}{10}$.	15. $15\frac{5}{6} + 9\frac{7}{12}$.

16. A farmer sold two stacks of hay, in one of which there were $8\frac{1}{4}$ tons and in the other $6\frac{1}{3}$ tons. What was the weight of both?

17. From a pole $21\frac{1}{2}$ ft. long there was cut off $4\frac{5}{8}$ ft. What was the length of the part remaining?

18. A box is $2\frac{5}{6}$ ft. long and $1\frac{3}{8}$ ft. wide. What length of string will pass around it?

19. A dealer bought grain at $56\frac{3}{4}$c. a bushel and sold it at $66\frac{2}{3}$c. What was his gain a bushel?

20. A builder having a certain piece of work to do engaged two men by the hour. The first worked $8\frac{5}{12}$ hours and the second $6\frac{4}{5}$ hours. For how many hours' work did the builder pay?

21. A lady bought three pieces of ribbon. The first contained $\frac{2}{3}$ of a yd., the second $3\frac{1}{4}$ yd., and the third $4\frac{5}{8}$ yd. How many yards of ribbon did she buy?

22. A farmer having 220 bu. of wheat sold $75\frac{5}{8}$ bu., sowed $26\frac{1}{8}$ bu., and used $12\frac{4}{5}$ bu. for flour. How many bushels had he left?

MULTIPLICATION OF A FRACTION BY A WHOLE NUMBER

How much is 3 times 2 dollars? 3 times 2 feet? 3 times 2 apples? 3 times 2 sevenths?

3 times 2 sevenths $= \frac{2}{7} \times 3$.

What, then, is the value of $\frac{2}{7} \times 3$? of $\frac{2}{3} \times 7$? of $\frac{5}{8} \times 4$?

How can we multiply a fraction by a whole number?

Example: Multiply $\frac{5}{6}$ by 4.

$$\frac{5}{6} \times 4 = \frac{20}{6} = 1\frac{0}{3} = 3\frac{1}{3}.$$

Instead of dividing the 20 and the 6 by 2 we find it more convenient, before multiplying, to divide the 4 and the 6 by 2. This is really dividing the numerator and the denominator by 2.

Thus $$\frac{5}{\not{6}_{3}} \times \frac{2}{\not{4}} = \frac{10}{3} = 3\frac{1}{3}.$$

This cancelling (see page 55) is not necessary in order to find the product, but it *reduces the product to its lowest terms.* The product is $\frac{20}{6}$, but this is equivalent to $1\frac{0}{3}$.

Multiply $\frac{21}{8}$ by 12. What, then, is the value of $2\frac{5}{8} \times 12$?

Since all mixed numbers can be changed to improper fractions, how then can they be multiplied by whole numbers?

The multiplication can also be performed as follows: $3\frac{1}{3} \times 6 = (3 \times 6) + (\frac{1}{3} \times 6) = 18 + 2 = 20$.

Example: Multiply $8\frac{2}{25}$ by 35.

$$8\frac{2}{25} \times 35 = \frac{202}{\cancel{25}_{5}} \times \overset{7}{\cancel{35}} = \frac{1414}{5} = 282\frac{4}{5};$$

or $(8 \times 35) + (\frac{2}{25} \times 35) = 280 + \frac{14}{5} = 280 + 2\frac{4}{5} = 282\frac{4}{5}$.

EXERCISE 44

Doing the work mentally, where possible, find the value of:

1. $\frac{1}{4} \times 4$.
2. $\frac{3}{4} \times 8$.
3. $\frac{3}{5} \times 7$.
4. $\frac{5}{9} \times 6$.
5. $\frac{7}{12} \times 9$.
6. $\frac{4}{15} \times 10$.
7. $\frac{7}{8} \times 80$.
8. $\frac{3}{10} \times 5$.
9. $\frac{5}{7} \times 21$.
10. $\frac{9}{25} \times 100$.
11. $\frac{7}{8} \times 24$.
12. $\frac{4}{49} \times 63$.
13. $3\frac{2}{3} \times 6$.
14. $8 \times 5\frac{1}{2}$.
15. $12 \times 2\frac{3}{4}$.
16. $22 \times 6\frac{5}{11}$.
17. $24 \times 3\frac{5}{16}$.
18. $20 \times 9\frac{4}{5}$.
19. $28 \times 10\frac{7}{8}$.
20. $4\frac{5}{24} \times 36$.

21. A wheat field containing 16 acres yields an average of $23\frac{3}{4}$ bu. an acre. Find the total number of bushels.

22. At $27\frac{2}{3}$ cents an hour how much will a man earn in 6 days, by working 9 hours a day?

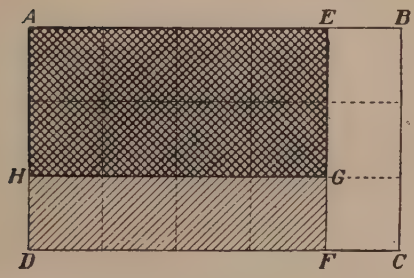
23. What is the value of $12\frac{5}{8}$ tons of coal at \$7·20 a ton?

24. At $\$5\frac{3}{5}$ a bu. what will be the cost of 8 bu. of clover seed?

25. A boy lives $1\frac{3}{8}$ miles from school. How far will he walk in one week in going to school in the morning and home in the evening?

26. John has $2\frac{3}{4}$ doz. eggs, and Fred has 3 times as many. Fred gives John $3\frac{1}{2}$ doz. How many will each then have?

COMPOUND FRACTIONS



Point out $\frac{1}{8}$ of the figure *ABCD*.
Point out $\frac{4}{8}$ of the same figure.
Point out $\frac{1}{2}$ of the figure *AEFD*.
Point out $\frac{2}{3}$ of the same figure.
Then point out $\frac{2}{3}$ of $\frac{4}{8}$ of the figure *ABCD*.

Now *AEGH* is divided into how many equal parts? How many of these equal parts are there in the whole figure *ABCD*?

Then *AEGH* is what fraction of *ABCD*?

Hence $\frac{2}{3}$ of $\frac{4}{8}$ is equal to what single fraction?

By drawing figures like the above, show how to get a single fraction for $\frac{1}{2}$ of $\frac{1}{3}$, for $\frac{3}{4}$ of $\frac{5}{8}$.

Without using a figure, state to what single fraction $\frac{4}{7}$ of $\frac{8}{5}$ is equal.

How then can a *fraction of a fraction*, that is, a compound fraction, be changed to a single fraction?

Example 1: Simplify $\frac{10}{21}$ of $\frac{14}{15}$.

$$\frac{10}{21} \text{ of } \frac{14}{15} = \frac{\overset{2}{\cancel{10}} \times \overset{2}{\cancel{14}}}{\underset{3}{\cancel{21}} \times \underset{3}{\cancel{15}}} = \frac{4}{9}.$$

Here again the cancelling is done before multiplying, in order that the answer may be given in its lowest terms.

Example 2: Find the value of $\frac{4}{9}$ of $\frac{6}{7}$ of $\frac{5}{8}$.

$$\frac{4}{9} \text{ of } \frac{6}{7} \text{ of } \frac{5}{8} = \frac{4}{9} \text{ of } \frac{6 \times 5}{7 \times 8}$$

$$= \frac{\overset{1}{\cancel{4}} \times \overset{3}{\cancel{6}} \times 5}{\underset{3}{\cancel{9}} \times 7 \times \underset{2}{\cancel{8}}} = \frac{5}{21}.$$

It will be found more convenient to do the work thus:

$$\frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}} \text{ of } \frac{\overset{3}{\cancel{6}}}{7} \text{ of } \frac{5}{\underset{2}{\cancel{8}}} = \frac{5}{21}.$$

By what have the numerator and denominator here been divided? Does that change the result? Why?

When mixed numbers occur, what change must first be made in them?

EXERCISE 45

Find the value of:

1. $\frac{1}{2}$ of $\frac{1}{3}$.
2. $\frac{1}{5}$ of $\frac{1}{6}$.
3. $\frac{1}{4}$ of $\frac{1}{8}$.
4. $\frac{1}{8}$ of $\frac{6}{7}$.
5. $\frac{1}{8}$ of $\frac{24}{25}$.
6. $\frac{1}{10}$ of $\frac{80}{91}$.
7. $\frac{3}{5}$ of $\frac{4}{7}$.
8. $\frac{7}{8}$ of $\frac{5}{9}$.
9. $\frac{3}{10}$ of $\frac{9}{11}$.
10. $\frac{1}{12}$ of $\frac{8}{15}$.
11. $\frac{1}{10}$ of $\frac{5}{7}$.
12. $\frac{2}{3}$ of $\frac{9}{10}$.
13. $\frac{7}{9}$ of 27.
14. $\frac{12}{25}$ of 40.
15. $\frac{11}{20}$ of 36.
16. $\frac{9}{14}$ of $3\frac{1}{2}$.
17. $\frac{10}{11}$ of $6\frac{3}{5}$.
18. $\frac{5}{8}$ of $9\frac{1}{11}$.
19. $\frac{4}{5}$ of $\frac{3}{7}$ of $\frac{8}{9}$.
20. $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{9}{16}$.
21. $\frac{15}{28}$ of $\frac{21}{25}$ of $\frac{12}{13}$.
22. $\frac{8}{11}$ of $\frac{3}{4}$ of $2\frac{1}{6}$.
23. $\frac{3}{16}$ of $\frac{24}{35}$ of $5\frac{1}{4}$.

24. Find the value: In pints, of $\frac{5}{16}$ of a bushel; in minutes, of $\frac{7}{9}$ of a week; in inches, of $\frac{3}{11}$ of a rod; in pence, of $\frac{3}{8}$ of a pound.

25. Find the value of:

(*a*) $\frac{4}{5}$ of 3 gal. 3 qt. 1 pt.

(*b*) $\frac{8}{9}$ of 7 yd. 2 ft. 8 in.

(*c*) $\frac{5}{6}$ of £9 2s. 6d.

26. A man sold $\frac{1}{3}$ of his flock of sheep and then $\frac{1}{2}$ of the remainder. What fraction of the whole flock had he left?

27. A miner owned $\frac{3}{8}$ of a mine. He sold $\frac{2}{3}$ of his share. What part of the mine did he sell and what part of the mine had he left?

28. A man owned a city lot containing $2\frac{4}{5}$ acres, and sold $\frac{3}{7}$ of it. How many acres did he sell?

29. John spent $\frac{7}{10}$ of his week's wages and lost $\frac{4}{9}$ of the remainder. What part of his wages did he lose? What part of his wages had he left?

30. Of a 45 mile journey $\frac{3}{7}$ is by rail, $\frac{3}{4}$ of the remainder is by boat, and the rest by coach. What is the distance by rail, by boat, and by coach?

31. Of a farm of 200 acres, $\frac{5}{8}$ is pasture and there is $\frac{3}{10}$ as much woodland as pasture. How many acres are there of woodland?

32. In a school of 350 pupils $\frac{1}{7}$ are in Form IV, $\frac{1}{6}$ of the remainder are in Form III, $\frac{3}{5}$ of the number then remaining are in Form II, $\frac{5}{8}$ of the number still remaining are in Form I, and the rest are in the Kindergarten. How many pupils are there in each Form and in the Kindergarten?

DIVIDING A FRACTION BY A WHOLE NUMBER

If a quantity is divided by 3, the quotient will be what part of the dividend?

If then the dividend is $\frac{6}{11}$ and the divisor is 3, the quotient will be what part of the dividend?

Then $\frac{6}{11} \div 3 = \frac{1}{3}$ of $\frac{6}{11} = \frac{2}{11} = \frac{6 \div 3}{11}$.

So also $\frac{5}{7} \div 4 = \frac{1}{4}$ of $\frac{5}{7} = \frac{5}{28} = \frac{5}{7 \times 4}$.

In what two ways may a fraction be divided by a whole number?

ORAL EXERCISE

Find the value of:

1. $\frac{8}{9} \div 2$.
2. $\frac{10}{13} \div 5$.
3. $\frac{21}{25} \div 7$.
4. $\frac{3}{8} \div 3$.
5. $\frac{5}{8} \div 4$.
6. $\frac{7}{10} \div 6$.
7. $2\frac{1}{2} \div 5$.
8. $3\frac{1}{5} \div 8$.
9. $6\frac{3}{4} \div 9$.
10. $1\frac{2}{3} \div 10$.
11. $5\frac{1}{3} \div 12$.
12. $4\frac{1}{2} \div 6$.

EXERCISE 46

Find the value of:

1. $\frac{24}{25} \div 16$.
2. $\frac{15}{26} \div 12$.
3. $8\frac{8}{9} \div 20$.
4. $9\frac{1}{11} \div 40$.
5. $724 \div 24$.
6. $315 \div 30$.

7. Divide the sum of $\frac{2}{3}$ and $1\frac{3}{5}$ by 6.

8. Divide the difference between $\frac{3}{8}$ and $\frac{7}{12}$ by 10.

9. A man who owns $\frac{3}{4}$ of a section of land divides it into 5 farms of equal size. What part of the section is in each farm?

10. Find the average of the following sums of money: $\$\frac{3}{5}$, \$2, $\$\frac{1}{2}$, $\$\frac{7}{8}$, $\$5\frac{1}{4}$, $\$8\frac{3}{10}$.

11. If 8 yd. of cloth cost $\$2\frac{4}{5}$. what was the price a yard?

12. A man sowed $11\frac{2}{3}$ bushels of oats on a 5-acre field. What was the average an acre? How many bushels at the same rate would be required for 3 acres?

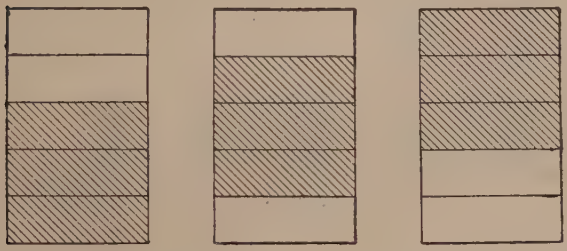
13. A piece of ground containing $2\frac{2}{3}$ acres was divided equally into 6 lots. How many acres would there be in 4 of the lots?

14. If for 9 yd. of cotton a lady pays $\$\frac{3}{8}$, what would she pay for 16 yd. at the same rate?

15. A man undertakes to drive $38\frac{1}{2}$ miles. He takes $3\frac{1}{5}$ hrs. to go the first 16 miles. How long will it take him to drive the remaining distance at the same rate?

16. How many pieces $\frac{5}{8}$ of an inch long can be cut from a wire 2 ft. 5 in. in length? What is the length of the part remaining?

MULTIPLYING A FRACTION BY A FRACTION



Examine these figures to see if they are equal. Is the shaded portion of one equal to that of each of the others? What part of each figure is shaded? If the figures were joined together, how many pieces, each equal to the shaded part of one of the figures, could be made out of the whole three figures together?

Then the shaded part of one of the figures is equal to what part of all the figures together?

$\frac{3}{5}$ is therefore equal to what part of 3?

Draw figures like the above to show that $\frac{2}{3} = \frac{1}{3}$ of 2, and that $\frac{4}{5} = \frac{1}{5}$ of 4.

Now $3 \div 5 = \frac{1}{5}$ of 3,
and $\frac{3}{5} = \frac{1}{5}$ of 3;
therefore $\frac{3}{5} = 3 \div 5$.

It is thus seen that a fraction is a **quotient.** (See page 91.)

What fraction of 12 is 4? How may 4 times a number be obtained from 12 times that number?

Multiply 56 by 12 and from the answer show how to find the value of 56×4.

$\frac{4}{5}$ is what fraction of 4? If $\frac{5}{7}$ be multiplied by 4, how can the value of $\frac{5}{7} \times \frac{4}{5}$ be found from the answer?

Example 1: Multiply $\frac{7}{10}$ by $\frac{8}{21}$.

$$\frac{7}{10} \times 8 = \frac{7 \times 8}{10},$$

therefore

$$\frac{7}{10} \times \frac{8}{21} = \frac{7 \times 8}{10} \div 21,$$

$$= \frac{\overset{1}{\cancel{7}} \times \overset{4}{\cancel{8}}}{\underset{5}{\cancel{10}} \times \underset{3}{\cancel{21}}} = \frac{4}{15}.$$

It is more convenient to do the work thus:

$$\frac{\overset{1}{\cancel{7}}}{\underset{5}{\cancel{10}}} \times \frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{21}}} = \frac{4}{5 \times 3} = \frac{4}{15}.$$

EXERCISE 47

Find the value of:

1. $\frac{5}{7} \times \frac{3}{4}$.
2. $\frac{8}{11} \times \frac{4}{9}$.
3. $\frac{9}{16} \times \frac{8}{9}$.
4. $\frac{4}{7} \times \frac{21}{16}$.
5. $\frac{1}{25} \times \frac{10}{21}$.
6. $\frac{13}{20} \times \frac{15}{26}$.
7. $\frac{8}{15} \times \frac{25}{36}$.
8. $\frac{27}{32} \times \frac{44}{45}$.
9. $\frac{18}{35} \times \frac{14}{27}$.
10. $2\frac{1}{8} \times \frac{15}{22}$.
11. $6\frac{2}{3} \times \frac{9}{10}$.
12. $\frac{16}{21} \times 8\frac{3}{4}$.
13. $5\frac{4}{9} \times 2\frac{4}{7}$.
14. $9\frac{2}{7} \times 3\frac{8}{13}$.
15. $8\frac{2}{3} \times 4\frac{1}{2}$.
16. $\frac{4}{7} \times \frac{5}{9} \times \frac{21}{40}$.
17. $6\frac{3}{9} \times \frac{25}{42} \times 2\frac{5}{8}$.
18. $2\frac{1}{3} \times 3\frac{1}{9} \times 4\frac{2}{7}$.
19. $2\frac{5}{6} \times \frac{4}{15} \times 1\frac{11}{34}$.
20. At $\$4\frac{4}{5}$ a yard what will $15\frac{5}{8}$ yd. of cloth cost?
21. At $\$3\frac{3}{4}$ a hundredweight what will $2\frac{1}{2}$ hundredweight of flour cost?
22. A surveyor drew a map of a city so that 1 inch on the map represented $\frac{1}{8}$ of a mile in the city. How many miles are there between two streets which are $3\frac{1}{8}$ in. apart on the map?

23. Make up the following bill for dress goods:

$3\frac{1}{2}$ yd. flannel at $\$\frac{3}{4}$ a yd.
$\frac{1}{6}$ doz. spools silk at $\$1\frac{1}{8}$ a doz.
$\frac{2}{3}$ doz. buttons at $\$\frac{9}{10}$ a doz.
2 pieces silk tape at $\$\frac{3}{20}$ each.

24. Make out the cost of the following cake recipe:

$\frac{9}{16}$ lb. butter at 24c. a lb.
1 lb. sugar at 6c. a lb.
$\frac{1}{4}$ qt. milk at 6c. a qt.
$\frac{7}{12}$ doz. eggs at 30c. a doz.
$\frac{5}{8}$ lb. flour at 3c. a lb.
$1\frac{1}{4}$ lb. seeded raisins at 15c. a lb.
1 lb. almonds at 10c. a lb.
$\frac{3}{4}$ lb. dates at 12c. a lb.
$\frac{1}{4}$ lb. citron at 10c. a lb.

25. A buyer bought $37\frac{1}{2}$ bu. of grain at $62\frac{1}{2}$c. a bushel. He sold $\frac{1}{3}$ of it at 70c. and the remainder of it at $65\frac{1}{2}$c. a bushel. What was his total gain?

26. How far will a man go in $2\frac{1}{3}$ hours if he travels at the rate of $8\frac{2}{5}$ miles an hour?

27. From the sum of $4\frac{2}{3}$ and $3\frac{1}{8}$ take their difference, and multiply the remainder by $4\frac{4}{5}$.

28. If one horse eats $\frac{1}{8}$ bu. of oats three times a day, how many bushels will be eaten by 9 horses in two weeks?

DIVISION BY A FRACTION

How many times is 20c. contained in \$2? 9 in. in 3 ft.? 3 qt. in 6 gal.? 3 fourths in 5 halves? 5 sixths in 7 thirds? 2 threes in 7 ninths?

Example 1: Divide $\frac{4}{5}$ by $\frac{3}{8}$.

$$\frac{4}{5} \div \frac{3}{8} = \frac{32}{40} \div \frac{15}{40} = 32 \text{ fortieths} \div 15 \text{ fortieths}$$
$$= 32 \div 15 = \frac{32}{15} = 2\frac{2}{15}.$$

State what has been done here.

Example 2: Divide 5 by $\frac{2}{5}$.

$$5 \div \tfrac{2}{5} = \tfrac{25}{5} \div \tfrac{2}{5} = 25 \div 2 = \tfrac{25}{2} = 12\tfrac{1}{2}.$$

Explain this.

Example 3: Divide $3\frac{3}{8}$ by $2\frac{1}{4}$.

$$3\tfrac{3}{8} \div 2\tfrac{1}{4} = \tfrac{27}{8} \div \tfrac{18}{8} = 27 \div 18 = \tfrac{27}{18} = \tfrac{3}{2} = 1\tfrac{1}{2}.$$

Explain this.

NOTE: $\frac{4}{5} \div \frac{3}{8} = \frac{32}{15}$, but $\frac{4}{5} \times \frac{8}{3} = \frac{32}{15}$;
therefore $\frac{4}{5} \div \frac{3}{8} = \frac{4}{5} \times \frac{8}{3}$.

Now does $5 \div \frac{2}{5} = 5 \times \frac{5}{2}$, and does
$\frac{27}{8} \div \frac{9}{4} = \frac{27}{8} \times \frac{4}{9}$?

How then can a fraction be divided by a fraction?

EXERCISE 48

Find the value of:

1. $\frac{1}{2} \div \frac{3}{4}$.
2. $\frac{5}{6} \div \frac{1}{2}$.
3. $\frac{4}{9} \div \frac{2}{3}$.
4. $\frac{5}{8} \div \frac{5}{6}$.
5. $\frac{11}{24} \div \frac{7}{8}$.
6. $\frac{9}{16} \div \frac{5}{12}$.
7. $\frac{7}{15} \div \frac{9}{10}$.
8. $2\frac{1}{4} \div \frac{3}{8}$.
9. $2\frac{2}{3} \div \frac{4}{15}$.
10. $3\frac{1}{7} \div 2\frac{1}{5}$.
11. $5\frac{3}{8} \div 7\frac{1}{6}$.
12. $6\frac{1}{4} \div 2\frac{1}{2}$.
13. $7\frac{1}{7} \div 4\frac{1}{6}$.
14. $14\frac{1}{5} \div 1\frac{3}{11}$.
15. $4\frac{3}{8} \div 9\frac{2}{7}$.
16. $6 \div \frac{3}{5}$.
17. $20 \div \frac{5}{8}$.
18. $9 \div \frac{6}{7}$.
19. $12 \div \frac{3}{4}$.
20. $8 \div \frac{3}{10}$.
21. $16 \div \frac{8}{9}$.
22. $25 \div 3\frac{3}{4}$.
23. $8\frac{2}{3} \div 2\frac{1}{6}$.
24. $62\frac{1}{2} \div 6\frac{1}{4}$.
25. $\frac{9}{24} \div \frac{5}{16}$.
26. $\frac{14}{15} \div \frac{11}{12}$.
27. $\frac{17}{52} \div \frac{51}{52}$.
28. $(\frac{4}{9}$ of $\frac{5}{11}) \div (\frac{8}{11}$ of $\frac{5}{18})$.
29. $5\frac{2}{5} \div (\frac{7}{9}$ of $\frac{5}{21})$.

30. At $\$\frac{3}{8}$ a yard how many yards of ribbon can be bought for $\$6\frac{1}{2}$?

31. If for $\frac{3}{4}$ of an acre of land there is paid $\$37\frac{1}{2}$, what is the price for an acre?

32. At the rate of $3\frac{2}{3}$ miles an hour how long will it take a man to walk $11\frac{1}{3}$ miles?

33. How many bags will be required for $31\frac{1}{2}$ bu. of grain if each bag will hold $2\frac{1}{4}$ bu.?

34. If 1 yd. of carpet covers $\frac{5}{6}$ of a sq. yd. of floor, how many yards of carpet will it take to carpet a floor containing 125 sq. yd.?

35. If $\frac{9}{10}$ of a pound of tobacco cost $\$\frac{1}{5}$, what will be the cost of 1 lb.? of $2\frac{1}{2}$ lb.?

36. If $\frac{7}{8}$ of a pound of coffee cost 35c., what should be paid for $\frac{9}{16}$ of a pound?

37. If there are $5\frac{1}{2}$ yd. in 1 rd., how many rods are there in 66 yd.? in $37\frac{1}{2}$ yd.? in 90 yd.?

38. A man's wages are $\$2\frac{3}{4}$ a day, out of which he spends $\$1\frac{1}{3}$ a day. In how many days can he save $\$58\frac{1}{2}$?

39. Two men $40\frac{1}{3}$ miles apart travel toward each other, one at the rate of $3\frac{1}{2}$ miles an hour, and the other at the rate of $3\frac{1}{3}$ miles an hour. In what time will they meet?

COMPARISON OF QUANTITIES

ORAL EXERCISE

1. What fraction is 1 of 2? of 3? of 5? of 10? of 15?

2. What fraction is 2 of 4? of 8? of 12? of 20? of 3? of 5?

3. What fraction is 4 of 12? of 16? of 28? of 6? of 22? of 9?

4. What fraction is 10 of 20? of 50? of 25? of 12? of 16? of 11? of 13? of 9? of 6?

5. What fraction is 12 of 36? of 60? of 15? of 16? of 18? of 22? of 17? of 7? of 8? of 6?

6. What fraction of \$6 is \$2? of 5 ft. is 3 ft.? of 10 lb. is 4 lb.? of 12c. is 5c.? of 7 eighths is 3 eighths? of 8 tenths is 5 tenths? of 12 twentieths is 9 twentieths? of $\frac{2}{3}$ is $\frac{4}{5}$?

7. What fraction of 1 yd. is 2 ft.? of 2 gal. is 3 qt.? of \$1 is 20c.? of 12 da. is 8 hr.? of $1\frac{1}{2}$ ft. is 7 in.

8. Express:

(*a*) 3 da. 12 hr. as a fraction of 2 wk.

(*b*) 10s. 6d. as a fraction of £3.

(*c*) 1 gal. 2 qt. 1 pt. as a fraction of 4 bu.

(*d*) 2 ft. 9 in. as a fraction of 6 ft. 3 in.

(*e*) 4 lb. 8 oz. as a fraction of 10 lb. 12 oz.

9. Express:

(*a*) $\frac{5}{6}$ of 8 yd. as a fraction of $\frac{3}{8}$ of 4 rd.

(*b*) $\frac{5}{8}$ of 9s. 4d. as a fraction of $\frac{2}{3}$ of £4 10s.

(*c*) $\frac{3}{8}$ of 1500 lb. as a fraction of $\frac{3}{4}$ of 2 tons 8 cwt.

10. What fraction of 5 eighths is 1 fourth? of 7 tenths is 3 fifths? of $\frac{5}{6}$ is $\frac{1}{2}$? of $\frac{5}{9}$ is $\frac{2}{3}$?

11. What fraction of $\frac{2}{3}$ is $\frac{3}{5}$? of $\frac{5}{7}$ is $\frac{1}{2}$? of $\frac{7}{12}$ is $\frac{5}{8}$? of $2\frac{1}{2}$ is $1\frac{1}{2}$? of $3\frac{1}{3}$ is $2\frac{1}{2}$? of 5 is $\frac{5}{8}$? of 7 is $\frac{3}{10}$?

12. What fraction of 6 yd. is 4 yd.? The cost of 4 yd. will be what fraction of the cost of 6 yd.? If 6 yd. cost 75c., what will 4 yd. cost?

13. If 8 ac. of land yield 124 bu. of grain, what will 6 ac. yield?

14. If in 20 min. a typewriter writes 825 words, how many words would be written in 15 min. at the same rate?

15. What fraction of 5 is $1\frac{1}{4}$? If a farmer ploughs $1\frac{1}{4}$ ac. of land in 2 days, how long will it take him to plough 5 ac. at the same rate?

16. A boy's wages for 12 hr. were \$2·46. What would be his wages for 10 hr. at the same rate?

17. If this line (————————) represents a school attendance of 300 pupils, draw lines to represent attendances of 100 pupils, 350 pupils, 600 pupils, and 850 pupils respectively.

EXERCISE 49

MISCELLANEOUS PROBLEMS

1. A house is 26 ft. long, its width is $\frac{5}{8}$ of its length. What is its perimeter?

2. A man who spent $\frac{7}{10}$ of his money found that he had $\$17\frac{1}{2}$ left. How much had he at first, and how much did he spend?

3. One third of the length of a pole is painted white, $\frac{3}{8}$ red, and the rest, 14 ft., is blue. What is the length of the pole, and of the part painted red?

4. A farmer sells $\frac{2}{5}$ of his farm and gives his son $\frac{1}{2}$ of the remainder. In what he now has there are 75 ac. How many acres were there in the farm at first?

5. By selling cloth at \$1·50 a yard a merchant loses $\frac{1}{8}$ of the cost. For what should he have sold it to gain $\frac{1}{10}$ of the cost?

6. Two boys have between them \$45·50. If one boy has $\frac{3}{4}$ of what the other boy has, find the amount each has.

7. A mixture containing $33\frac{1}{2}$ lb. of corn, $25\frac{5}{8}$ lb. of oats, and $17\frac{3}{4}$ lb. of barley will feed how many cows, if each is given $5\frac{1}{8}$ lb.?

8. A man mixes together oats, corn, and barley so that there are 4 lb. of oats for every 3 lb. of corn and 2 lb. of barley. What fraction of the mixture is barley? How much barley will there be in a mixture weighing 450 lb.?

9. From $232\frac{1}{2}$ yd. of cotton $185\frac{2}{3}$ yd. were sold. What will the remainder be worth at $10\frac{1}{2}$c. a yd.?

10. Make out a bill for the following:

555 lb. of wheat @ 78c. a bu.
289 lb. of oats @ 43c. a bu.
432 lb. of peas @ 55c. a bu.
304 lb. of timothy seed @ \$2·60 a bu.
469 lb. of corn @ 64c. a bu.
472 lb. of buckwheat @ 54c. a bu.

11. Mr. Thomas borrows some money and agrees to pay for the use of it, each year, $\frac{1}{20}$ of the sum borrowed. What would he pay for the use of \$125 for 1 yr.? for 4 yr.?

12. If for the use of money I pay each year $\frac{1}{25}$ of the sum borrowed, what should I pay for the use of \$225 for $3\frac{1}{3}$ yr.?

13. Mr. Jones and Mr. Turner are partners in a business in which the former invested \$2000 and the latter \$4000. What fraction of the business does each own? How should a profit of \$750 be divided between them?

14. How many mills are there in a dollar? 20 mills is what fraction of \$1? If a man pays a tax of 20 mills on every dollar he is worth and his total tax is \$75, what is he worth?

15. A man sold 240 bu. of wheat, which was $\frac{4}{7}$ of all he had. What would be the value of $\frac{1}{2}$ of what he had left, at $87\frac{3}{8}$c. a bushel?

16. A woman sold $49\frac{1}{2}$ lb. of butter at $28\frac{2}{3}$c. a pound, and with the money purchased cotton at $6\frac{2}{3}$c. a yard. How many yards of cotton did she buy?

17. If it takes A 5 days to do a piece of work, what fraction of the work does he do in 1 day? If B does the same work in 3 days, what fraction of the work does he do in 1 day? What fraction of the work would A and B together do in 1 day? How many days would it take A and B together to do the work?

18. A does a piece of work in 6 days, and B can do the same work in 4 days. How many days would it take A and B together to do the work?

19. A carpenter works $55\frac{1}{3}$ hr. at $22\frac{1}{2}$c. an hour. If out of this he has to pay for 15 meals at the rate of 6 meals for \$1, how much will he have left?

20. If it takes a boy $3\frac{1}{2}$ hr. to walk 7 mi., how long will it take him to walk $8\frac{1}{2}$ mi. at the same rate?

21. Divide 45 marbles between 2 boys so that one will have 4 times as many as the other.

22. Divide \$62 between two men so that one will have \$10 more than 3 times as much as the other.

23. A merchant sold $\frac{4}{9}$ of his goods at a profit of $\frac{1}{4}$ of their cost, and the remainder at a profit of $\frac{1}{10}$ of their cost. His whole gain was \$150. What was the total cost of the goods?

24. Arrange the following fractions in order of value:

$$\frac{3}{4},\ \frac{7}{10},\ \frac{5}{6},\ \frac{17}{20},\ \frac{23}{30}.$$

25. From the sum of all the fractions in Question 24 take the difference between the greatest and the least.

26. A certain contractor agrees to build a house for \$2250, and is to receive at the end of each week $\frac{4}{9}$ of the value of the work completed during the week. At the end of the eighth week he has finished $\frac{7}{9}$ of his contract. What amount of money should he have received by that date?

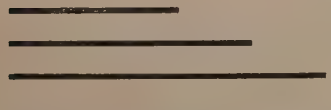
27. If it requires $25\frac{7}{8}$ bu. of grain to sow $11\frac{1}{2}$ ac., how many bushels at the same rate will be required for $9\frac{3}{4}$ ac.?

28. At a certain hour of the day a pole $7\frac{1}{2}$ ft. long casts a shadow 10 ft. long. What is the length of a pole the shadow of which is $6\frac{2}{3}$ ft. long?

29. A merchant beginning business invested $\frac{1}{2}$ his money in dry goods, $\frac{1}{4}$ in groceries, and $\frac{1}{12}$ in hardware, and had left \$1500 for other supplies. How much money had he at first?

30. A newsdealer bought papers at the rate of 4 for 5c. and sold them at the rate of 3 for 5c. His gain was 65c. How many papers did he sell?

31. These lines represent a merchant's sales for September, October, November and December. If the first line represents sales amounting to \$8000, find the total sales for the four months, using a rule.



32. A man divided some land among his four children as follows: to the eldest he gave 240 acres, to the second $\frac{1}{5}$ of the whole property, to the third $\frac{1}{6}$ of the whole property, and to the youngest as much as to the second and third together. What did each receive, and what was the size of his estate?

PRACTICAL MEASUREMENTS

(See pages 71-74)

Draw, to a scale $\frac{1}{4}$ in. to 3 ft., the wall of a house 33 ft. long, 21 ft. high, and show a door, at the centre of the ground line, $7\frac{1}{2}$ ft. high and 3 ft. wide. Show also two windows, each 3 ft. wide and 6 ft. high, placed $4\frac{1}{2}$ ft. from the ground line and 6 ft. on either side of the door.

Draw a rectangle 4 inches long and 3 inches wide. Divide the length and width into inches and join the opposite points.

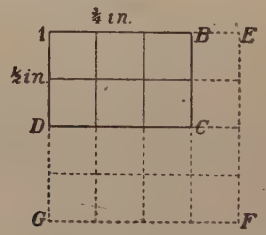
The whole rectangle is divided into how many parts? What is the shape of each of these parts? What is its size? Into what square units, then, is the rectangle divided? How many square units are there in the rectangle?

This may be expressed thus:

Area = 3 times 4 square inches, or 3×4 times 1 square inch.

The length and the width of a surface are called its **dimensions.**

Now, if the linear units in the *dimensions* had been feet, what would the square unit have been? if yards? if rods?



Again, *ABCD* is a rectangle $\frac{3}{4}$ in. long and $\frac{1}{2}$ in. wide.

Divide the length and width into the same linear units, namely, $\frac{1}{4}$ in. Join opposite points.

The figure *ABCD* is divided into how many parts? What is the shape of each of these parts? How many of these

parts are there in the dotted figure? What is the size of the dotted figure? The figure $ABCD$ is then what part of the dotted figure?

This may be expressed thus:

Area of $ABCD = 2 \times 3 \times \frac{1}{16}$ sq. in.

$= \frac{6}{16}$ or $\frac{3}{8}$ or $\frac{1}{2} \times \frac{3}{4}$ sq. in.

If the $\frac{3}{4}$ and the $\frac{1}{2}$ were feet, what would the area be? If rods?

Draw a figure, to a scale, to show how to find the area of a rectangle 10 rods long by 6 rods wide. Also of one $2\frac{1}{2}$ yd. long by $1\frac{1}{2}$ yd. wide.

The number of square units in the area of any rectangle is thus equal to the number of linear units in the length, multiplied by the number of linear units in the width.

Observe that the length and the width must be given in units of the same denomination; that is, both must be inches, or both feet, or both yards, etc.

If the area of a rectangle and one of its dimensions be given, how can the other dimension be found?

State a short method for finding the area of a square, and also one for finding its perimeter.

ORAL EXERCISE

1. What is the area of a rectangle 8 in. long and 6 in. wide? What is its perimeter?

2. What is the area of a rectangle $\frac{3}{4}$ ft. long and $\frac{2}{3}$ ft. wide?

3. What is the area of a rectangle $\frac{5}{6}$ ft. long and 5 in. wide? What is its perimeter?

4. How many square yards are there in the surface of a piece of paper 1 yd. long and $\frac{5}{4}$ yd. wide?

5. How many square yards are there in the area of a piece of carpet 1 yd. long and 27 in. wide?

6. How many cards 3 in. long and 3 in. wide can be cut from a card 9 in. long and 6 in. wide?

7. What is the area of a square 9 in. in length? What is its perimeter?

8. The perimeter of a square is 20 in. What is its area?

9. What will be the width of a rectangle whose area is 18 sq. in. and whose length is 6 in.?

10. What is the perimeter of a plot of ground whose area is 36 sq. yd. and the length of which is 4 yd.? How many yards of wire will it require to go five times round it?

11. How many pieces of paper 1 yd. long and $\frac{3}{4}$ yd. wide will it require to cover a bulletin board 6 yd. long and 3 yd. wide?

12. How much will be the cost of a piece of tin 4 ft. long and 3 ft. wide at 3c. a square foot?

13. How many square yards are there in the surface of a black-board 15 ft. long and 3 ft. wide? How much will it cost to paint it at 20c. a sq. yd.?

14. How much will it cost to paint a wall 18 ft. long and 3 yd. high at 10c. a sq. yd.?

In solving problems in land measuring, painting, plastering, papering, carpeting, roofing, shingling, and such like, it will be found helpful to draw plans of grounds, buildings, walls, etc., to a scale.

Instead of stating that a surface is 20 ft. long and 15 ft. wide, it is customary to say that the surface is 20 ft. by 15 ft., or more briefly still, $20' \times 15'$.

EXERCISE 50

1. How many acres are there in rectangular fields: 38 rd. by 20 rd.? 48 rd. by 30 rd.? 60 rd. by 32 rd.? 80 rd. by 40 rd.?

2. Find the dimensions of your school ground and find the number of acres in it.

3. For a farm 240 rd. long and 144 rd. wide a man paid $45 an acre. What did the farm cost him ?

4. A city bought a street 66 ft. wide and 180 rd. long at $100 an acre. How much did the street cost ?

5. At $11\frac{1}{2}$c. a sq. ft. what will be the cost of building a cement walk $\frac{1}{4}$ mi. long and $4\frac{1}{2}$ ft. wide ?

6. Around the outside of a block of land 40 rd. by 20 rd. there is a cement walk 4 ft. wide. What did the walk cost at 15c. a sq. ft.?

7. Find the width of a plot of ground which contains 6 acres and which is 60 rd. long. Find also its perimeter.

8. How many yards of wire will be required to build a fence 5 wires high around a 12-acre field which is 60 rd. long ?

9. At $1\frac{1}{2}$c. a sq. ft. what will it cost to sod a strip of ground 10 rd. long and 36 ft. wide ?

10. Find the cost of painting the floor of a room 24 ft. by 18 ft. at 20c. a sq. yd.

11. A man builds a board fence 6 ft. high around his lot which is 100 ft. by 50 ft. What is the total length of the fence ? What will it cost to paint both sides of it at 15c. a sq. yd.?

12. How many cards 2 in. by $1\frac{1}{2}$ in. can be cut from 50 sheets of bristol board, each sheet being 27 in. by 22 in.? What will the cards cost at 6c. a 100 ?

13. Around a two-acre lot which is 20 rd. long, the owner builds a wire fence with posts 22 ft. apart. How many posts will be required ?

14. How many square inches of cardboard will be required to make a candy box 6 in. long, 4 in. wide, and 2 in. deep, no allowance being made for overlapping ?

15. Cut out the pattern for the candy box given in Question 14, and find the number of square inches which will actually be required by your plan in making the box with parts folded so as to hold it together.

16. Find, by drawing a pattern, the number of square inches of cardboard which will be required to make a box 8 in. long, 4 in. wide, and 3 in. deep.

17. If a shingle covers 4 in. by $3\frac{1}{2}$ in. of a roof, how many shingles will be required for the two sides of a roof of a barn, each side of the roof being 35 ft. by 20 ft.?

18. How much will it cost for shingles for the roof of a building, each side of the roof being 42 ft. by 25 ft., if the shingles cost $5 a thousand?

NOTE: Shingles are sold by the 1000. A thousand shingles are supposed to cover one square of roof, a square being 100 sq. ft.

19. If shingles are 4 in. wide and are laid 5 in. to the weather, how many hundreds will be required for a roof 40 ft. long and 16 ft. 8 in. from eaves to ridge?

20. Find the cost of kalsomining the walls and ceiling of a room 21 ft. long, 16 ft. wide, and 9 ft. high at 4c. a sq. yd., deducting a window 3 ft. by $4\frac{1}{2}$ ft. and a door 3 ft. by $7\frac{1}{2}$ ft.

21. Find the cost of kalsomining the walls and ceiling of your class-room at 6c. a sq. yd.

22. At 35c. a sq. yd. find the cost of plastering the room given in Question 20, deducting *one half* of the area of the openings.

NOTE: In plastering, the allowance made for openings is a matter of arrangement with the workman, but it is usual to deduct one half of the area of the openings.

23. At 30c. a sq. yd. find the cost of plastering the walls and ceiling of a room 18 ft. long, 14 ft. wide and 6 ft. above the wainscot, deducting one half the area of one window 3 ft. by 5 ft. and of one door 7 ft. by 3 ft.

24. A ploughman ploughs a furrow 1 ft. wide and 40 rd. long every 3 minutes. How long will it take him to plough 1 acre at the same rate?

25. At 60c. a sq. yd. find the cost of a tin roof each side of which is 36 ft. by 22 ft. 6 in.

26. There are two square plots of ground, the length of one being twice that of the other. If it costs $20 to sod the smaller of the two, what will it cost at the same rate to sod the larger?

27. A plank is 10 ft. long and 10 in. wide. How many square inches are there in its upper surface?

28. A board is 12 ft. long. How wide must it be in order that its upper surface may measure 1 sq. ft.? 2 sq. ft.? 3 sq. ft.? 10 sq. ft.? any number of square feet?

CARPETING AND PAPERING

Carpet and wall-paper are made in strips, and the purchaser must buy strips of full width.

Cover the top of your desk with strips of paper of equal width. The total width of all the strips is equal to what other width? What two widths would you require to know in order to find out how many strips would be needed? Make all the strips into one long strip. How long will it be?

Draw a plan of a floor and show how carpet would be laid upon it. How could the number of strips required be found? How could the total length of all the strips be found? (See page 53.)

In some carpets where patterns must be matched, it may be necessary to purchase more carpet than is actually used.

This will apply to wall-paper also. The number of strips required, the length of each strip, and the amount of paper wasted in matching patterns must all be known.

Wall-paper is sold in *single* rolls of 8 yards and *double* rolls of 16 yards, and it is usually either 18 in. or 21 in. in width.

EXERCISE 51

1. A floor 18 ft. by 15 ft. is covered with carpet 27 in. wide running lengthwise. How many strips are there? How many yards long is each strip? How many yards of carpet are there altogether? What would the carpet cost at $1·20 a yard?

2. If the carpet in the preceding question were 30 in. wide and laid lengthwise, what would it cost? If laid across the floor, what would it cost?

3. Which way must the strips of a carpet 36 in. wide run in order to make a saving in the cost of carpet for a floor 22 ft. long and 18 ft. wide? What would the carpet cost at $2 a yard?

4. How many yards of carpet 33 in. wide will it require to cover a floor 16 ft. 6 in. by 21 ft. 2 in.?

5. If there is a carpet pattern every 15 in. in the length of a carpet, how many times will the pattern be repeated in one strip running lengthwise on a floor 18 ft. long? How many inches will be wasted in each of the other strips in matching?

6. How many yards of the carpet in Question 5 will be required if the floor is 11 ft. 3 in., and the carpet 27 in. wide?

7. How many yards of carpet 30 in. wide will it take to carpet a floor 13 ft. 6 in. long and 10 ft. wide, if 6 in. of each strip after the first is wasted in matching? What will the carpet cost at $1·25 a yard?

8. The walls of a room 21 ft. long, 15 ft. wide, and 8 ft. high are papered with paper 18 in. wide, and no allowance is made for openings. How many strips of paper will be required? how many yards will be required? how many single rolls? What will the paper cost at 30c. a roll?

9. How many yards of wall-paper 18 in. wide will be required to cover the walls of a room 15 ft. by 12 ft. by 8 ft., making no allowance for openings?

10. Find the cost of paper 21 in. wide for the ceiling of a room 16 ft. by 10 ft. 6 in. at 10c. a yard.

11. The walls of a room 18 ft. long, 15 ft. wide and 8 ft. high above the baseboard are papered with paper 18 in. wide. Find the cost of paper at 40c. a single roll.

12. How many single rolls of wall-paper 18 in. wide will be required for the four walls of a room 22 ft. 6 in. long, 18 ft. wide and 11 ft. high, there being a wainscot 4 ft. 4 in. high all round the room?

BOARD MEASURE

The unit used in the measurement of lumber is the **board foot,** by which is meant the amount of lumber contained in a board 1 ft. long, 1 ft. wide, and 1 in. thick.

Make such a board. What is the area of its upper or lower surface? How often is this area contained in the area of one of the surfaces of a board 12 ft. long, 12 in. wide? in the area of one of the surfaces of a board 12 ft. long and 8 in. wide? in the area of one of the surfaces of a board 6 ft. long and 6 in. wide?

A board 12 ft. long, 12 in. wide and 1 in. thick contains 12 *board feet.* A board 12 ft. long, 8 in. wide and 1 in. thick contains 8 *board feet*; and a board 6 ft. long, 6 in. wide, and 1 in. thick contains 3 *board feet.*

If each of the above boards were 2 in., $2\frac{1}{2}$ in. or $1\frac{1}{4}$ in. thick, it would contain 2, $2\frac{1}{2}$ or $1\frac{1}{4}$ times as many board feet.

Lumber is bought and sold by the *board foot*, and the price is usually quoted at so much "per M," which means per 1000 ft. board measure.

Boards less than one inch thick are generally sold as inch lumber.

The dimensions of a board are given as follows:

$$11' \times 8'' \times 1\tfrac{1}{4}''$$

which means 11 ft. long, 8 in. wide, and $1\frac{1}{4}$ in. thick.

ORAL EXERCISE

Find the number of board feet in boards 1 in. thick and whose other dimensions are:

1. $8' \times 4''$.
2. $9' \times 3''$.
3. $8' \times 3''$.
4. $9' \times 4''$.
5. $10' \times 12''$.
6. $15' \times 12''$.
7. $10' \times 6''$.
8. $4' \times 3''$.
9. $6' \times 4''$.
10. $12' \times 9''$.
11. $12' \times 6''$.
12. $12' \times 5''$.
13. $12' \times 7''$.
14. $12' \times 11''$.

Find the number of board feet in planks or scantlings whose dimensions are:

15. $12' \times 6'' \times 2''$.
16. $12' \times 3'' \times 4''$.
17. $12' \times 5'' \times 2''$.
18. $12' \times 2'' \times 2''$.
19. $12' \times 5'' \times 4''$.
20. $12' \times 6'' \times 1\frac{1}{2}''$.
21. $12' \times 10'' \times 2\frac{1}{2}''$.
22. $12' \times 8'' \times 1\frac{1}{4}''$.
23. $12' \times 6'' \times 4''$.

In the preceding oral exercises (numbers 13 to 23), it will be seen that the number of board feet in a board 12 ft. long is equal to the number of *inches* in the *width* of the board multiplied by the number of *inches* in its *thickness.*

Thus, a board 12 ft. long, 8 in. wide and $2\frac{1}{2}$ in. thick contains $2\frac{1}{2} \times 8$ bd. ft., or 20 bd. ft.

Now, an 8-ft. board contains $\frac{8}{12}$ as many board feet as a 12-ft. board of the same width and thickness. So, also, a 15-ft. board will contain $\frac{15}{12}$ as many board feet as a 12-ft. board of like width and thickness.

EXERCISE 52

Find the number of board feet in boards, planks, or scantlings of the following dimensions:

1. $15' \times 8'' \times 1\frac{1}{4}''$.
2. $14' \times 10'' \times 1\frac{1}{2}''$.
3. $8' \times 6'' \times 4''$.
4. $16' \times 15'' \times 4\frac{1}{2}''$.
5. $10'' \times 5'' \times 3\frac{1}{4}''$.
6. $9' \times 3\frac{1}{2}'' \times 4\frac{1}{2}''$.
7. $12' \times 5'' \times 2\frac{1}{4}''$.
8. $16' \times 2\frac{1}{2}'' \times 2\frac{3}{4}''$.
9. $14' \times 4'' \times 3''$.

10. How much lumber is there in a stick 24 ft. long, 15 in. wide, and 12 in. thick?

11. A man bought 24 planks 15 ft. long, 18 in. wide, and $3\frac{1}{2}$ in. thick. How much did they cost at 6c. a board foot?

12. What will be the cost of 30 pieces of pine scantling $12' \times 4'' \times 5''$ at \$40 per M?

13. What will be the cost of inch maple for a hardwood floor for a room 25 ft. by 20 ft. at \$65 per M?

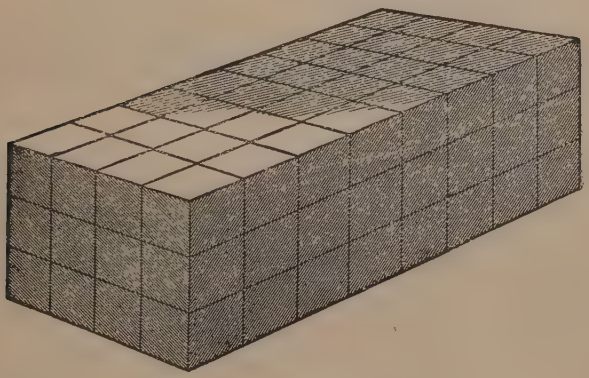
14. How many board feet are there in a solid pile of lumber 10 ft. long, 20 ft. wide and 8 ft. high?

15. How much inch lumber 6 in. wide will it take to build a fence 5 boards high around a lot 60 yd. long by 40 yd. wide?

16. How much lumber $1\frac{1}{2}$ in. in thickness will it require to build a board side-walk 4 ft. wide and $\frac{1}{8}$ of a mile long, the boards being nailed to 3 scantlings $4'' \times 4''$?

17. How much inch lumber will it require to make a covered box the outside dimensions of which are 20 in. by 1 ft. by 6 in.? (Make a drawing of this.)

MEASURE OF VOLUME OR CAPACITY



The solid figure shown above is called a **rectangular block** or **solid.**

If this block were 4 inches wide, 3 inches high, and 8 inches long, what would be the size of one of the small blocks into which it is divided? If the dimensions were given in feet, what then would one of the small blocks be?

With inch cubes build up a block like the figure here shown. How many cubes wide is the block? How many

cubes high? How many cubes long? How many cubes are there in the whole block?

When you find the number of cubic units contained in any solid you are said to find its **volume** or **capacity.**

The volume or capacity of any solid figure is the amount of space inclosed within its bounding faces. It is measured by the number of **cubic units** it contains.

Just as rectangular *surfaces* can be divided into *squares*, so rectangular *solids* can be divided into *cubes.* How?

Look again at the figure on the preceding page. Into how many slices is it cut? How many rows are there in each slice? How many cubes in each row?

Its volume $= 4 \times 3 \times 8$ times 1 cubic unit
$= 96$ cubic units.

Therefore, the number of cubic units in the volume of any rectangular solid is equal to the number of linear units in its length, multiplied by the number of linear units in its width, multiplied by the number of linear units in its height.

All the linear units must be of the same *denomination.*

If the number of cubic units in the volume and the number of linear units in two of its dimensions be given, how can the number of linear units in the third dimension be found?

ORAL EXERCISE

Find the number of cubic inches in rectangular blocks whose dimensions are:

1. $3'' \times 2'' \times 1''$.
2. $4'' \times 3'' \times 2''$.
3. $6'' \times 5'' \times 3''$.
4. $10'' \times 8'' \times 5''$.

Find the number of cubic feet in rectangular solids whose dimensions are:

5. $9' \times 5' \times 3'$.
6. $8' \times 6' \times 3''$.
7. $6' \times 24'' \times 12''$.
8. $4' \times 18'' \times 8''$.

Find the volume of rectangular solids whose dimensions are:

9. $8'' \times 5'' \times 4''$.
10. $6' \times 12'' \times 8''$.
11. $2' \times 18'' \times 4''$.
12. $3' \times 4' \times 6''$.

Find the third dimension when the volume is:

13. 36 cu. in. and the other dimensions are $4''$ and $3''$.

14. 84 cu. in. and the other dimensions are $7''$ and $6''$.

15. How many cubic inches of lumber are there in 1 board foot?

16. How many cubic inches of earth can be put in a box $8'' \times 4'' \times 3''$?

17. How many cubic yards of earth must be removed to leave a rectangular hole $9'$ by $3'$ by $2'$?

EXERCISE 53

1. Find the volume (in cubic feet) of a room 22 ft. long, 18 ft. wide and 10 ft. high.

2. Find the number of cubic yards of gravel which will be required to gravel a piece of road 40 rd. long and 6 ft. wide if the gravel is laid on 18 in. deep. What will the gravel be worth at 60c. a cubic yard?

3. How many cubic feet are there in a cement walk 30 ft. long and $4\frac{1}{2}$ ft. wide and 15 in. deep?

4. How many cubic feet of water will it take to fill a rectangular cistern $4\frac{1}{2}$ ft. long, $3\frac{1}{2}$ ft. wide and 8 ft. deep?

5. If there are 450 cu. ft. in a ton of hay, how many tons can be put in a mow $40'$ by $25'$ by $13\frac{1}{2}'$?

6. A ton of coal measures about 40 cu. ft. How many tons can be put into a bin 12 ft. $\times$ 7 ft. $\times$ 6 ft.?

7. At 40c. a cubic yard how much will it cost to dig a cellar $27' \times 16' \times 9'$?

8. How many gallons of water will it take to fill a rectangular tank $10' \times 6' \times 4'$?

9. From a trench 3 ft. wide and 2 ft. deep 80 cu. yd. of earth were taken. How long was the trench?

10. In the wall of a room is a ventilator $2' \times 1\frac{1}{2}'$. How many feet of air must pass through it in order that 3600 cu. ft. of air may enter the room?

11. At what rate a minute must the air flow through the ventilator in Question 10 in order that the amount required may enter the room in 5 minutes? What will be the rate a second?

12. A room is $20' \times 15' \times 8'$. At what rate a second must air pass through a ventilator $2\frac{1}{2}' \times 2'$ in order that there may be a complete change of air every 4 minutes?

13. Through a rectangular opening 20 ft. wide and 4 ft. deep a stream flows at the rate of 3 miles an hour. How many cubic feet of water will pass through the opening in 10 minutes? How many gallons would flow through in the same time, assuming $6\frac{1}{4}$ gallons in a cubic foot?

14. How many bricks $8'' \times 4'' \times 2\frac{1}{2}''$ will it take to build a solid brick wall without mortar, 30 ft. long, 4 ft. high and $1\frac{3}{4}$ ft. wide?

15. How many cubic feet of masonry will be required for the foundation of a house, $44' \times 30'$, the foundation wall being 10 ft. high and 2 ft. wide?

16. With a rainfall of 2 in. how many gallons of water would fall on an acre of land if there are $6\frac{1}{4}$ gallons in a cubic foot?

EXERCISE 54

1. Read aloud the following: In 1907, the expenditure for primary schools in Ontario was \$7556179, an increase of \$1152973. The amount paid for teachers' salaries was \$4389524, an increase of \$508976. The aggregate attendance was 448218. Out of 64001 visits, 10958 were paid by trustees, 3647 by clergymen, 16882 by inspectors, and 32514 by other persons. The legislative grant to High Schools was \$87060, and the municipal grant, \$112062. In the Collegiate Institutes, of 16148 pupils 10279 were in the Lower School and 1437 in the Upper.

2. A coal dealer delivered at a school 14 loads of coal of which the weights were: 4220 lb., 4280 lb., 4170 lb.,

3831 lb., 4600 lb., 4680 lb., 4630 lb., 4580 lb., 4790 lb., 2610 lb., 3350 lb., 4800 lb., 4480 lb., 4370 lb.

Find in pounds, and also in tons, the total amount of coal delivered. Find also the average weight in pounds of the loads.

3. Divide 236847 by 64, using factors 4, 4, and 4.

4. At $16800 a mile what will it cost to build a railway from Montreal to Fort William, a distance of 995 miles?

5. A dealer paid $62·25 for a number of bags of potatoes. If he had bought 7 more bags he would have paid $67·50. How many bags did he buy?

6. It costs to send a telegram 25c. for the first 10 words, and 1c. for each word above 10. What will it cost to send a telegram containing 87 words?

7. Make out a bill for the following groceries: $4\frac{1}{2}$ lb. butter @ 30c., $2\frac{1}{4}$ doz. eggs @ 28c., $7\frac{3}{4}$ lb. coffee @ 36c., 3 lb. tea @ 65c., 5 lb. cheese @ $12\frac{1}{2}$c., 3 lb. biscuits @ 25c., $12\frac{1}{2}$ lb. sugar @ 20 lb. for $1·00.

8. A freight car is 36 ft. long, 8 ft. 4 in. wide, and 8 ft. high. How many bushels of wheat will it hold, if full, assuming that there are $1\frac{1}{4}$ cu. ft. in one bushel?

9. If the railway company allows the above car to carry only 75000 lb., how many bushels too much would there be in the car when full of wheat?

10. A dry-goods merchant sells 75 yd. of cloth for $126·50, thereby gaining $14. What did the cloth cost him a yard?

11. This pay roll shows the number of hours a day each man works. Fill in the wages at 28c. an hour and check your work.

	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Wages
W. B. Jones	8	$7\frac{1}{2}$	9	$8\frac{1}{4}$	9	5	
T. R. Henry	9	9	0	8	$6\frac{1}{2}$	0	
S. R. Lock	7	$8\frac{1}{2}$	8	$7\frac{3}{4}$	$8\frac{1}{2}$	5	
D. M. Black	$7\frac{1}{4}$	8	$8\frac{1}{4}$	$8\frac{1}{2}$	$7\frac{3}{4}$	$4\frac{1}{2}$	

12. An express train travelling at an average rate of 35 miles an hour leaves Toronto at 10·15 p.m. for Montreal, a distance of about 336 miles. At what time should it arrive?

13. On New Year's Day a gentleman made a resolution to spend during the year not more than $500 for personal expenses. On 30th September he found he had already spent $385. How much a day, on an average, can he spend for the rest of the year to keep within his resolution?

14. Find the cost of the following materials for a gown: 9 yd. worsted @ 75c., $\frac{1}{2}$ yd. silk at $2, $1\frac{1}{2}$ yd. silk piping @ $1, 2 spools silk thread @ 10c., and $2\frac{3}{4}$ yd. lawn @ 12c.

15. A man travelled $\frac{1}{3}$ of a certain distance by boat, $\frac{2}{3}$ by train, and walked $\frac{1}{4}$ of the remaining distance. If he walked $3\frac{1}{2}$ miles, what was the total distance?

16. It was reported that during a storm the wind was blowing at the rate of 45 miles an hour. What would this rate be in feet a second?

17. During the month of March, 1910, Mr. Brace deposited in, and drew from, his bank the following amounts: 1st Mar. had in bank $522·64; 3rd Mar. deposited $96·24; 10th Mar. deposited $164·58; 12th Mar. drew $112·92; 15th Mar. deposited $81·00; 16th Mar. drew $97·87; 21st Mar. drew $20·04; 26th Mar. deposited $95·83; 30th Mar. drew $9·75.

Rule a page of a bank-book to show the balance after each transaction and at the end of the month.

18. A farmer has a cow which gives, on an average, 6 qt. 1 pt. of milk daily. If he sells the milk at $5\frac{1}{2}$c. a quart, how much will he receive for it from the 1st June to 30th Sept., both dates included?

19. How many feet of lumber $1\frac{1}{4}$ in. thick will it require for the floor of a room 20 ft. long and 16 ft. wide? What will it cost at $50 a thousand feet?

20. Divide $186 among three men, giving the second twice as much as the first, and the third as much as the other two.

21. If it costs $6·75 to plough $3\frac{3}{4}$ acres of land, what will it cost at the same rate to plough $9\frac{1}{2}$ acres?

22. A lot containing $\frac{5}{8}$ of an acre was sold at \$12 a foot frontage. If its frontage measured 82 ft. 6 in., what was the price an acre?

23. For proper lighting of a school-room the window space should measure $\frac{1}{6}$ of the floor space. Does your school-room supply the necessary amount of light?

FACTORS

What is meant by factor? prime factor? composite number?

The prime factors of a number can be found only by trial. The composite factors of a number may be found either (1) by trial, or (2) by finding the prime factors of the number and then multiplying two or more of these prime factors together.

ORAL EXERCISE

1. What are the factors of 12? of 16? of 28?

2. Name five prime numbers, five composite numbers.

3. Which of the following numbers are prime: 6, 11, 17, 21, 23, 39, 47, 51?

4. Name all the composite numbers between 18 and 42.

5. Of what numbers is 5 a factor? 2 a factor?

6. Of what number are 2, 3, and 5 the factors?

7. Of what number are 2, 2, 2, 2, and 2 the factors?

8. What are the prime factors of 6? of 15? of 20? of 27? of 36? and of 45? Name *all* the factors of each of these numbers.

9. Which of the following numbers will 3 divide: 718, 324, 6587, 3855, 42368? (A number is divisible by 3 when the sum of its digits is divisible by 3.)

10. Which of the numbers in Question 9 will 9 divide? (A number is divisible by 9 when the sum of its digits is divisible by 9.)

Example: Find the prime factors of 2520.

We set down the work thus:

Explanation: We divide by 2, the *first prime* number. When 2 no longer divides exactly we divide by 3, the *second prime* number. When 3 no longer divides exactly we divide by 5, the *third prime* number, etc.

$$\begin{array}{r} 2)2520 \\ 2)1260 \\ 2)630 \\ 3)315 \\ 3)105 \\ 5)35 \\ 7 \end{array}$$

The required prime factors are 2, 2, 2, 3, 3, 5, and 7; and $2520 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$.

EXERCISE 55

Express each of the following numbers as the product of its prime factors:

1. 324.	5. 420.	9. 882.	13. 6293.	17. 13041.
2. 336.	6. 495.	10. 1962.	14. 7995.	18. 18291.
3. 390.	7. 605.	11. 4335.	15. 1875.	19. 19800.
4. 392.	8. 840.	12. 5390.	16. 12447.	20. 36498.

21. Find the prime factors of 1296 and combine these factors so as to get two equal factors.

22. Find the prime factors of 1728 and combine these so as to get three equal factors.

23. Find the prime factors of 4080 and combine these so as to get for factors three consecutive numbers.

24. What is the length, in yards, of the side of a square plot of ground containing 729 square yards?

HIGHEST COMMON FACTOR

What numbers divide both 12 and 18? What is the greatest number that divides 12 and 18?

A number that divides two or more numbers exactly is called a **common factor**, a **common measure** or a **common divisor.**

The greatest factor common to two or more numbers is called their **highest common factor** (H.C.F.), or **greatest common measure** (G.C.M.).

When two numbers have no *common factor* greater than 1, they are said to be **prime to each other.**

ORAL EXERCISE

1. Name all the factors of 18 and of 24.

2. Name all the common factors of 18 and 24.

3. Name their *highest common factor.*

4. Name all the factors, all the common factors, and the highest common factor of 28 and 42.

5. Name all the factors, all the common factors, and the highest common factor of 6, 8, 10.

6. How, now, can you find the highest common factor of two or more numbers?

7. What is the highest common factor of 9 and 15? of 16 and 36? of 12, 18, and 30?

8. Name two numbers which are *prime to each other.*

9. From the following, select pairs of numbers which are prime to each other: 7, 9, 10, 15, 14, 16, 21.

10. From the list of numbers in Question 9, select pairs of numbers which are not prime to each other.

Name all the common factors of 48 and 64. Which is the greatest?

Again using prime factors:

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

and their H.C.F. $16 = 2 \times 2 \times 2 \times 2$.

Therefore there is another method of finding the H.C.F. of two or more numbers. What is it?

The work may be set down thus:

$$\begin{array}{r|l} 2)48 & 64 \\ 2)24 & 32 \\ 2)12 & 16 \\ 2)6 & 8 \\ 3 & 4 \end{array}$$

The H.C.F. $= 2 \times 2 \times 2 \times 2 = 16$.

EXERCISE 56

Find the highest common factor of:

1. 45 and 75.
2. 153 and 374.
3. 272 and 425.
4. 13, 52 and 91.
5. 60, 84 and 96.
6. 455 and 728.
7. 928 and 1073.
8. 1536 and 3584.
9. 2037 and 6598.
10. 2233 and 4147.
11. 2108 and 3813.
12. 3056 and 3629.
13. 112, 128 and 192.
14. 182, 221 and 299.
15. 165, 220 and 385.

16. Find the greatest whole number by which both 738 and 954 can be exactly divided.

17. What is the length of the longest cord which will exactly measure both the length and width of a rectangular lot which is 154 yd. long and 132 ft. wide? How many times the length of the cord is the length of the perimeter of the lot?

MULTIPLES

When one number contains another exactly, the former is said to be a **multiple** of the latter. What number is a *multiple* of 7? of 3? of 6?

A number which is exactly divisible by each of two or more numbers is called a **common multiple** of those numbers. Name a *common multiple* of 4 and 6; of 8 and 12; of 3, 4, and 5.

The **least common multiple** (L.C.M.) of two or more numbers is the smallest number which will exactly contain each of them. What is the L.C.M. of 2, 3, and 5?

ORAL EXERCISE

1. Name the first 6 *multiples* of 5, of 7, of 3.

2. Name the first 8 multiples of 4, and of 6.

3. Name the first 3 *common multiples* of 4, and 6. What, therefore, is the L.C.M. of 4 and 6?

4. Name the L.C.M. of 5 and 3. Name their next two common multiples.

5. When the *least common multiple* of two or more numbers is found, how can other common multiples be found?

6. What must be the prime factors of the L.C.M. of 2, 3, and 5? Why? What must be the prime factors of the L.C.M. of 4, 7, and 9? Why? Then how can the L.C.M. of numbers which are prime to each other be found?

7. What is the L.C.M. of 2, 4, 6, and 8? Why is this the same as the L.C.M. of 6, and 8? What is the L.C.M. of 3, 5, and 10? Why is this the same as the L.C.M. of 3, and 10? Then in finding the L.C.M. of 3, 4, 8, and 9, of which of these numbers is it necessary to take account? Why?

Example 1: Find the L.C.M. of 24, 30, and 36:

$$24 = 2 \times 2 \times 2 \times 3.$$
$$30 = 2 \times 3 \times 5.$$
$$36 = 2 \times 2 \times 3 \times 3.$$

Now, it is evident that the L.C.M. of these three numbers must contain all their *different* prime factors, 2, 3, and 5, and that each of these factors must be repeated as many times, and no more, as it is repeated in that number in which it occurs the greatest number of times. Thus 2 must be used 3 times, 3 twice, and 5 once. Hence the required L.C.M. is $2 \times 2 \times 2 \times 3 \times 3 \times 5$ or 360.

The work may be set down thus:

2)24, 30, 36
2)12, 15, 18
3) 6, 15, 9
2, 5, 3

L.C.M. $= 2 \times 2 \times 3 \times 2 \times 5 \times 3$ or 360.

Example 2: Find the L.C.M. of all the multiples of 2 and 3 from 2 to 20, inclusive.

The numbers are: 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, and 20. Now, since 2, 3, 4, 6, 8, 9, and 10 are each contained in one or other of the remaining numbers, it is only necessary to find the L.C.M. of 12, 14, 15, 16, 18, and 20.

The work will then be as follows:

2)12, 14, 15, 16, 18, 20
2) 6, 7, 15, 8, 9, 10
3) ~~3~~, 7, 15, 4, 9, ~~5~~
7, 5, 4, 3

L.C.M. is $3 \times 4 \times 5 \times 7 \times 3 \times 2 \times 2 = 5040$.

In the third line 3 and 5 were struck out. Why?

EXERCISE 57

Find the L.C.M. of the following:

1. 16, 18, and 20.
2. 12, 28, and 64.
3. 27, 36, and 42.
4. 22, 25, 30, and 33.
5. 15, 30, 42, and 75.
6. 12, 14, 16, 32, 50.
7. 12, 15, 16, 18.
8. 22, 77, 143.
9. 8, 12, 16, 24, 36.
10. 5, 7, 9, 11, 15, 21, 27.
11. 36, 40, 45.
12. 26, 39, 51, 65.
13. 80, 64, 108, 720.
14. 36, 60, 65, 78.
15. 221, and 533.
16. 144, 186, 496.
17. 918, and 969.
18. 240, 480, and 960.

19. Four bells toll at intervals of 12, 14, 20, and 21 seconds respectively. If they all commence together, after what interval will they again toll together?

20. The circumference of the small wheel of a wagon is 9 ft., and that of the larger wheel is 12 ft. How often in going one mile will the points of the two wheels which were on the ground at starting be on the ground again at the same instant?

21. A man has a piece of ground the length of which is 770 yd., and the width 396 yd. He wishes to inclose it with a wire fence with posts a uniform distance apart, this distance to be the greatest possible. Find the distance between the posts. Find also the number of posts which will be needed. (*Draw a plan.*)

22. A number on being divided by either 7, 9, 12, 15, or 20, leaves in each case a remainder of 6. What is the number?

FRACTIONS

EXERCISES FOR DRILL

Most of the fractions in common use have been dealt with in a preceding section. With the aid of greatest common measure and least common multiple many of the operations with fractions may be made easier.

EXERCISE 58

1. What is a fraction? What is meant by the **terms** of a fraction? What is the use of each of the *terms?*

2. When is a fraction in its lowest terms? How are fractions reduced to their lowest terms? What factor is used when the reduction is made in a single step?

3. What is meant by saying that two fractions are equivalent? How is a fraction changed to an equivalent one? How many equivalents can a fraction have?

Supply numerators for each of the following:

4. $\frac{99}{121} = \frac{\quad}{11}$.
5. $\frac{108}{612} = \frac{\quad}{51}$.
6. $\frac{126}{216} = \frac{\quad}{12}$.
7. $\frac{525}{735} = \frac{\quad}{7}$.
8. $\frac{864}{1296} = \frac{\quad}{3}$.
9. $\frac{1001}{1617} = \frac{\quad}{21}$.
10. $\frac{3}{11} = \frac{\quad}{77}$.
11. $\frac{9}{17} = \frac{\quad}{255}$.
12. $\frac{6}{31} = \frac{\quad}{217}$.
13. $\frac{5}{7} = \frac{\quad}{252}$.
14. $\frac{13}{27} = \frac{\quad}{5643}$.
15. $\frac{43}{72} = \frac{\quad}{2232}$.

Supply denominators for each of the following:

16. $\frac{12}{67} = \frac{4}{\quad}$.
17. $\frac{65}{85} = \frac{13}{\quad}$.
18. $\frac{88}{1332} = \frac{22}{\quad}$.
19. $\frac{90}{216} = \frac{15}{\quad}$.
20. $\frac{231}{308} = \frac{21}{\quad}$.
21. $\frac{324}{612} = \frac{9}{\quad}$.
22. $\frac{5}{7} = \frac{125}{\quad}$.
23. $\frac{8}{11} = \frac{392}{\quad}$.
24. $\frac{21}{80} = \frac{147}{\quad}$.

Reduce each of the following fractions to its lowest terms:

25. $\frac{10}{44}$.
26. $\frac{34}{85}$.
27. $\frac{70}{119}$.
28. $\frac{84}{259}$.
29. $\frac{72}{272}$.
30. $\frac{315}{420}$.
31. $\frac{468}{604}$.
32. $\frac{384}{416}$.
33. $\frac{836}{976}$.
34. $\frac{630}{936}$.
35. $\frac{84}{1932}$.
36. $\frac{2288}{10024}$.
37. $\frac{8225}{27709}$.
38. $\frac{1242}{28328}$.
39. $\frac{1440}{17728}$.
40. $\frac{2508}{80386}$.
41. $\frac{1680}{43512}$.
42. $\frac{6400}{72000}$.

43. Read aloud all the fractions in this exercise.

EXERCISE 59

1. What is a proper fraction? What is a mixed number? To what fractions can mixed numbers be reduced? How? Is this process Reduction Ascending or Descending? Explain.

2. What must be done with fractions before they can be added or subtracted?

3. What is meant when it is said that two or more fractions have a **common denominator**? What *common denominator* is the most convenient for use? How can it be found?

Reduce to equivalent fractions having common denominators:

4. $\frac{2}{5}, \frac{2}{3}, \frac{4}{9}$.
5. $\frac{5}{7}, \frac{2}{5}, \frac{11}{21}$.
6. $\frac{5}{6}, \frac{2}{11}, \frac{3}{7}$.
7. $\frac{1}{6}, \frac{5}{12}, \frac{7}{18}$.
8. $\frac{5}{24}, \frac{11}{27}, \frac{13}{72}$.
9. $\frac{7}{9}, \frac{8}{45}, \frac{19}{75}$.
10. $\frac{5}{16}, \frac{7}{18}, \frac{9}{20}, \frac{13}{24}$.
11. $\frac{3}{14}, \frac{9}{25}, \frac{5}{28}, \frac{7}{30}, \frac{8}{35}$.

Arrange the following fractions in order of magnitude:

12. $\frac{5}{7}, \frac{5}{12}, \frac{11}{15}, \frac{20}{27}$.
13. $\frac{5}{26}, \frac{3}{52}, \frac{13}{65}, \frac{7}{39}$.

Add or subtract as indicated:

14. $\frac{24}{35} + \frac{19}{21} + \frac{11}{14}$.
15. $\frac{3}{8} + \frac{4}{5} + \frac{11}{20} + \frac{7}{12}$.
16. $\frac{7}{24} + \frac{7}{9} + \frac{11}{18}$.
17. $\frac{17}{30} + \frac{5}{32} + \frac{21}{40}$.
18. $\frac{7}{8} + \frac{4}{3} + \frac{3}{4} + \frac{5}{6} + \frac{2}{3}$.
19. $\frac{8}{10} + \frac{7}{11} + \frac{13}{20} + \frac{13}{35} + \frac{15}{44}$.
20. $\frac{15}{16} - \frac{7}{24}$.
21. $\frac{33}{26} - \frac{14}{65}$.
22. $\frac{17}{19} - \frac{7}{15}$.
23. $\frac{23}{30} - \frac{23}{40}$.
24. $\frac{31}{64} - \frac{13}{48}$.
25. $\frac{90}{121} - \frac{27}{66}$.
26. $\frac{12}{35} - \frac{4}{21} + \frac{7}{15}$.
27. $\frac{11}{12} - \frac{7}{36} - \frac{4}{27}$.
28. $\frac{61}{80} - \frac{3}{5} + \frac{13}{42} - \frac{7}{30}$.
29. $\frac{3}{11} + \frac{5}{22} - \frac{1}{5} - \frac{3}{10}$.
30. $\frac{5}{8} - (\frac{21}{27} - \frac{7}{18})$.
31. $\frac{5}{4} + (\frac{87}{75} - \frac{4}{15}) - \frac{11}{30}$.

Find the value of the following:

32. $2\frac{1}{30} + 1\frac{4}{15}$.
33. $7\frac{11}{15} + 9\frac{1}{27}$.
34. $5\frac{3}{16} - 2\frac{9}{40}$.
35. $11\frac{11}{35} - 7\frac{9}{14}$.
36. $22\frac{2}{9} + 17\frac{1}{2} + 5\frac{3}{5} + \frac{7}{8}$.
37. $34\frac{2}{3} + 14\frac{5}{12} - 13\frac{3}{16} - 1\frac{7}{24}$.
38. $9\frac{5}{51} - 5\frac{3}{9} + 11\frac{5}{21}$.
39. $6\frac{10}{11} - 2\frac{9}{22} - 1\frac{17}{20}$.
40. $14 - 3\frac{2}{3} + 5\frac{3}{5} - 1\frac{1}{2}$.
41. $6\frac{7}{45} - 2\frac{2}{7} + 4\frac{3}{35} - 3\frac{10}{63}$.

42. From $9\frac{3}{8} - 2\frac{5}{12}$, take the difference between $5\frac{7}{24}$ and $7\frac{3}{32}$.

EXERCISE 60

1. How are fractions of fractions simplified?

2. In this process what use is made of cancellation?

3. Fold a piece of paper in such a way that you can show: $\frac{3}{8}$ of $\frac{1}{4}$ of it, $\frac{3}{8}$ of $\frac{1}{2}$ of it, $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{3}$ of it.

4. Draw a line and mark on it $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{4}$ of the line.

Simplify:

5. $\frac{4}{5}$ of $\frac{1}{2}\frac{2}{3}$ of $3\frac{3}{8}$.
6. $\frac{25}{36}$ of $\frac{21}{65}$ of $\frac{39}{49}$.
7. $\frac{70}{81}$ of $\frac{51}{91}$ of $\frac{27}{40}$ of $5\frac{1}{7}$.
8. $\frac{5}{6}$ of $\frac{27}{55}$ of $\frac{10}{99}$ of $11\frac{1}{5}$.
9. $\frac{11}{39}$ of $\frac{26}{33}$ of $(4\frac{1}{8} - 2\frac{5}{8})$.
10. $\frac{17}{45}$ of $\frac{20}{61}$ of $(\frac{3}{4} + \frac{10}{17} - \frac{10}{34})$.
11. $\frac{21}{20}$ of $\frac{21}{20}$ of $\frac{21}{20}$ of \$800.
12. $\frac{24}{25}$ of $\frac{11}{10}$ of $\frac{9}{8}$ of \$750.

NOTE: Fractions connected by "of" must be taken together and simplified before being added to, or subtracted from, other fractions.

Find the value of:

13. $\frac{1}{12}$ of $\frac{8}{33} + \frac{2}{5}$ of $3\frac{3}{8} + \frac{1}{2}$ of $\frac{1}{3}$ of $4\frac{2}{7}$.

14. $\frac{35}{63}$ of $4\frac{1}{6} - \frac{26}{45}$ of $1\frac{2}{13} + \frac{7}{10}$ of $\frac{14}{15}$ of $8\frac{1}{8}$.

15. $3\frac{3}{4} - \frac{2}{5}$ of $2\frac{1}{2} + \frac{1}{3}$ of $2\frac{1}{4} - \frac{4}{7}$ of $2\frac{1}{8}$.

EXERCISE 61

1. How is the product found when a fraction is multiplied by a whole number? by a fraction? by a mixed number?

2. What use is made of cancellation in the multiplication of fractions?

Simplify:

3. $\frac{33}{44} \times \frac{33}{65}$.

4. $\frac{72}{86} \times \frac{68}{81}$.

5. $3\frac{5}{11} \times 1\frac{9}{57}$.

6. $3\frac{3}{4} \times 7\frac{11}{15}$.

7. $2\frac{1}{10} \times 4\frac{2}{7}$.

8. $3\frac{6}{19} \times 95$.

9. $\frac{9}{20} \times \frac{70}{81} \times \frac{27}{28}$.

10. $\frac{16}{65} \times \frac{11}{48} \times \frac{15}{17}$.

11. $\frac{18}{23} \times \frac{29}{45} \times \frac{115}{116}$.

12. $4\frac{1}{4} \times 5\frac{1}{5} \times 1\frac{3}{17}$.

13. $9\frac{5}{8} \times 1\frac{71}{121} \times 2\frac{45}{49}$.

14. $4\frac{23}{25} \times \frac{10}{41}$ of $2\frac{23}{69}$.

15. $\frac{6}{7}$ of $\frac{11}{12} \times \frac{9}{14}$ of $6\frac{1}{8}$.

NOTE: Quantities joined by the sign of multiplication must be multiplied together before being added to, or subtracted from, other quantities.

Find the value of:

16. $\frac{3}{4} \times \frac{4}{7} + \frac{3}{8} \times 1\frac{1}{2} + \frac{3}{7} \times 1\frac{1}{4}$.

17. $4\frac{3}{11} - 2\frac{3}{4} + 5\frac{1}{8} \times 4\frac{1}{2}$.

18. $(\frac{5}{8} + \frac{10}{11}) \times (\frac{3}{4} + \frac{7}{15}) \times \frac{73}{78} \times \frac{888}{2880}$.

19. $\frac{29}{105} + \frac{4}{35}$ of $1\frac{3}{4} - \frac{7}{11}$ of $\frac{6}{49} \times 3\frac{3}{8}$.

20. $7\frac{1}{2} \times (9\frac{1}{2} - 5\frac{1}{14}) - 2\frac{1}{7} \times 9\frac{1}{2} - 5\frac{1}{14}$.

21. Suggest business problems the answers for which would be found by solving examples 7, 8, 12, 16, and 17 in this exercise.

EXERCISE 62

1. How is a fraction divided by an integer? by a fraction? by a mixed number? How is a whole number divided by a fraction?

2. Give the multiplication questions which are the equivalent of the following questions in division:

$\frac{5}{8} \div \frac{2}{8}$, $\frac{4}{9} \div \frac{5}{6}$, $\frac{11}{15} \div \frac{4}{5}$, $2\frac{4}{5} \div 4\frac{4}{5}$, $7\frac{1}{3} \div 2\frac{3}{4}$, $1\frac{1}{5} \div \frac{3}{7}$.

3. Find the quotients for the exercises given in Question 2.

Simplify:

4. $18\frac{3}{4} \div 4\frac{1}{6}$.
5. $\frac{8}{15} \div \frac{4}{25}$.
6. $\frac{7}{12} \div 1\frac{4}{15}$.
7. $27\frac{1}{7} \div 1\frac{5}{14}$.
8. $93\frac{3}{4} \div 8\frac{1}{8}$.
9. $21 \div \frac{8}{5}$.
10. $9\frac{5}{7} \div 51$.
11. $18 \div \frac{6}{11}$.
12. $8\frac{1}{3} \div 75$.
13. $3\frac{17}{28} \div \frac{11}{161}$.
14. $7\frac{5}{16} \div 62\frac{1}{48}$.
15. $33\frac{3}{25} \div 2\frac{91}{265}$.

Explanation: Sometimes $3\frac{1}{5} \div 3\frac{3}{4}$ is written $\frac{3\frac{1}{5}}{3\frac{3}{4}}$. It is then called a **complex fraction,** that is, a fraction which has a fraction in its numerator, or in its denominator, or in both.

A fraction whose numerator and denominator are whole numbers is called a **simple fraction.**

Complex fractions are reduced to *simple* ones by changing them into exercises in division, the numerator being the dividend and the denominator the divisor, thus:

$$\frac{3}{\frac{4}{5}} = 3 \div \frac{4}{5}, \quad \frac{\frac{2}{7}}{4} = \frac{2}{7} \div 4, \quad \frac{5\frac{1}{3}}{2\frac{1}{5}} = 5\frac{1}{3} \div 2\frac{1}{5}, \text{ etc.}$$

Simplify:

16. $\dfrac{6\frac{3}{5}}{8}$.
17. $\dfrac{7\frac{1}{7}}{25}$.
18. $\dfrac{8\frac{3}{4}}{70}$.
19. $\dfrac{9}{4\frac{1}{2}}$.
20. $\dfrac{18}{7\frac{1}{5}}$.
21. $\dfrac{64}{9\frac{5}{11}}$.
22. $\dfrac{2\frac{1}{3}}{4\frac{1}{5}}$.
23. $\dfrac{8\frac{1}{3}}{5\frac{1}{5}}$.
24. $\dfrac{4\frac{1}{11}}{7\frac{7}{8}}$.
25. $\dfrac{\frac{2}{3} \text{ of } 1\frac{1}{5}}{1\frac{1}{2} - \frac{3}{8}}$.
26. $\dfrac{3\frac{1}{7} \times 2\frac{8}{33}}{1\frac{5}{6} - \frac{1}{2}}$.

Find the value of:

27. $(3\frac{7}{8} - 2\frac{1}{4}) \div (3\frac{1}{4} \times \frac{1}{2})$.

28. $(3\frac{1}{8} \times 2\frac{2}{5} \times 1\frac{3}{4}) \div 2\frac{1}{3}$.

29. $\frac{4}{5}\frac{9}{5}$ of $5\frac{3}{7} \div \frac{1}{2}\frac{9}{7}$ of $\frac{2}{4}\frac{4}{3}$.

30. $(3\frac{1}{3} - \frac{5}{6} + \frac{1}{5}) \div (2\frac{2}{3} + 1\frac{2}{5})$.

31. Make up questions the answers for which would be found by solving 9, 12, 20, 22, 27 in this exercise.

EXERCISE 63

In order that there may be no confusion in exercises where fractions are connected by the signs $+$, $-$, $\times$, $\div$, it is agreed that:

(*a*) Operations of multiplication and division are to be performed in the order in which they occur from left to right.

(*b*) Operations of multiplication and division are to be performed before those of addition and subtraction.

(*c*) Quantities within brackets are to be considered as a single quantity.

Simplify:

1. $\frac{1}{2}\frac{4}{6}$ of $\frac{1}{8}\frac{3}{6}$ of $7\frac{1}{8} \div 4\frac{1}{8} \times \frac{4}{8}\frac{4}{1}$.

2. $\frac{2}{1}\frac{9}{0}\frac{9}{5} + \frac{4}{8}\frac{}{5}$ of $\frac{1}{4}\frac{3}{} - \frac{7}{1}\frac{}{1}$ of $3\frac{2}{3} \div 8\frac{1}{6}$.

3. $(2\frac{1}{3} \times \frac{3}{1}\frac{}{4} - 5\frac{1}{2} + 16\frac{1}{2}) \div (16\frac{1}{2} - 5\frac{1}{2} + 1\frac{3}{1}\frac{}{4} + 2\frac{1}{8})$.

4. $5\frac{2}{5} - 3\frac{1}{5} \div 1\frac{1}{8} - 4 \div 1\frac{9}{1}\frac{}{1}$.

5. $(4\frac{3}{4} - 2\frac{5}{7}) \div 3\frac{4}{5} + 8 - 5\frac{1}{1}\frac{}{1} \times \frac{1}{4}\frac{1}{1}$.

6. $2\frac{1}{5} + 1\frac{4}{7} \div 1\frac{1}{2}\frac{}{1} - 2\frac{1}{7}$.

7. $\dfrac{3\frac{3}{4}}{4\frac{2}{7}} - \dfrac{3\frac{2}{5}}{4\frac{1}{4}} + \dfrac{\frac{4}{7}}{2\frac{1}{2}}$.

8. $(1\frac{2}{3} \times \frac{3}{5}$ of $21) \div 23$.

9. $\frac{3}{8}\frac{8}{1} \times \frac{4}{6}\frac{5}{6} \div \frac{2}{2}\frac{2}{7} \times \frac{1}{2}\frac{9}{1}$.

10. $5\frac{7}{8} \times 1\frac{3}{4}\frac{}{7} - 3\frac{3}{8} \div 2\frac{4}{7} - 1\frac{3}{1}\frac{}{8}$.

11. $1\frac{3}{8} \div (\frac{7}{9} \div 4\frac{1}{2}) \times 4\frac{2}{8}$.

12. $11\frac{1}{1}\frac{1}{2} \times 1\frac{8}{1}\frac{}{8} \div 2\frac{1}{4} + 7 - 3\frac{1}{9} \times 2\frac{1}{7} + \frac{1}{6} \div 2\frac{2}{8} \times \frac{5}{6}$.

EXERCISE 64

1. A man having \$1260 spent \$840 of it. What fraction remains of the money he had at first?

2. (*a*) Express 1 gal. 1 pt. as a fraction of 1 bu.

(*b*) If $6\frac{1}{2}$ bu. of wheat sell for \$5·20, for what would 1 gal. 1 pt. sell at the same rate?

3. Divide \$630 between two partners, giving one $\frac{1}{8}$ more than the other.

4. A farmer sells:

3460 lb. of wheat at 75c. a bu.
1785 lb. of oats at 48c. a bu.
6837 lb. of pease at 65c. a bu.
1468 lb. of pork at \$8·30 a cwt.

How much did he receive for all?

5. Find the average of the following amounts: $\$36\frac{2}{3}$, $\$44\frac{3}{8}$, $\$83\frac{3}{10}$, $\$92\frac{1}{6}$, $\$55\frac{7}{15}$.

6. A says he can do in 6 days a piece of work which B says he can do in 4. In what time should the two working together do the work at their own estimate?

7. A walnut board 13 ft. 6 in. long, 11 in. wide, and 2 in. thick, sells at $11\frac{1}{2}$c. a board foot. How many feet of pine at \$36 per M could one purchase for the same money?

8. Make out a bill for the following quantities of lumber at \$30 per M:

30 pieces measuring $12' \times 8'' \times 2''$
50 pieces measuring $10' \times 12'' \times 1''$
40 pieces measuring $12' \times 4'' \times 4''$
15 pieces measuring $16' \times 3'' \times 6''$
21 pieces measuring $8' \times 4'' \times 5''$.

9. (*a*) What fraction of the sum of $\frac{5}{6}$, $\frac{2}{3}$ and $\frac{7}{12}$ is $\frac{5}{6}$?

(*b*) Divide \$30 among three boys so that one will get $\$\frac{5}{6}$ as often as the second gets $\$\frac{2}{3}$ and the third gets $\$\frac{7}{12}$.

10. From $64\frac{3}{4}$ yd. of cloth a merchant sold to one customer $9\frac{1}{2}$ yd., to another $17\frac{5}{8}$ yd., to a third $8\frac{5}{8}$ yd., and to a fourth $15\frac{3}{8}$ yd. What will the remainder be worth at 60c. a yard?

11. A boy having a number of marbles divided them into three equal parts and lost one of these parts at play. He divided what were left into nine equal parts and gave away five of these parts. He then divided what were left into four equal parts and sold three of these parts. He had 8 marbles left. How many had he at first?

12. A piece of cloth, when measured by what was thought to be a yard-stick, was said to contain 24 yd. If the stick was $\frac{3}{4}$ of an inch too short, what was the actual length of the cloth?

13. If it takes $8\frac{1}{4}$ bu. of grain to sow $6\frac{7}{8}$ acres, how many acres will $4\frac{1}{2}$ bu. sow?

14. (*a*) What is the price, in mills, of one ℔. of hay when one ton costs \$12? \$10? \$8? \$7? \$9? \$8½? \$9¾? any sum in dollars?

(*b*) Using (*a*), find the cost of 270 ℔. of hay at \$8 a ton, of 3420 ℔. at \$7½ a ton, of 4330 ℔. of coal at \$6¼ a ton.

15. Divide \$364 among A, B, and C, giving A \$20 more than B, and B \$24 more than twice as much as C.

16. A merchant mixed 30 ℔. of tea which cost 35c. a ℔. with 80 ℔. which cost 56c. a ℔. What fraction of the cost would he gain by selling the mixture at 63c. a ℔.?

17. By working $8\frac{1}{4}$ hr. a day a man finished a piece of work in $10\frac{2}{3}$ days. How long should it have taken him had he worked $7\frac{1}{3}$ hr. a day?

18. A gentleman gave $\frac{1}{3}$ of his estate to his wife, $\frac{2}{3}$ of the remainder to his son, and the rest, \$4500, to his daughter. What was the total value of the estate?

19. Into a cistern which contains 101 gal., water is flowing steadily at the rate of $23\frac{3}{4}$ gal. an hour, but there is a leak through which are lost $5\frac{2}{7}$ gal. an hour. In how many hours will the cistern be filled?

20. Two trains are $81\frac{1}{4}$ miles apart and are running toward each other, one at the rate of $35\frac{1}{2}$ miles an hour and the other at the rate of $28\frac{3}{8}$ miles an hour. How far will they be apart in 40 minutes?

21. A newsdealer buys papers at the rate of 2 copies for 3c. and sells them to the newsboys at the rate of 3 copies for 5c. How many copies will he require in order that he may make a profit of \$1? How much will the newsboys make by selling the papers at 2c. each?

DECIMALS

Give the *place* name for each digit in 444444. How does each 4 differ in value from the 4 immediately to its right? from the second 4 on its right? from the 4 immediately to its left? from the second 4 on its left?

If a digit be put to the right of the units' digit, its *place* value should be what fraction of the place value of the units' digit? If a digit be placed two places to the right of the units' digit, its place value will be what fraction of the place value of the units' digit? Its place value would be what fraction of the place value of the digit to the right of the units' digit?

Hence digits to the right of the units' digit represent fractions. What would be the denominator of the fraction represented by the digit immediately to the right of the units' digit? of the fraction represented by the digit two places to the right of the units' digit? three places to the right?

Ordinary notation can now be extended to include fractions whose denominators are *10 or some power of 10*, that is 10, 100, 1000, 10000, etc.

Such fractions are called **decimal fractions** or simply **decimals.**

All other fractions are called **common** or **vulgar fractions.**

In order to show whether a digit represents an integer or a fraction, a dot (·) called the **decimal point** is placed to the right of the units' digit. All figures to the left of this point will represent integers, and all to the right, fractions.

Thus in 36·45, the 3 is tens, the 6 is units, the 4 is tenths, and the 5 is hundredths. The value of each digit depends, as in whole numbers, upon the place it occupies.

NOTATION AND NUMERATION

(See page 4, and Review Exercise 3.)

To read and write decimals the Table for ordinary numeration and notation is extended as follows:

Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
						3	1	2	4		
			5	6	7	0	8	9			

·3124 is read: Three thousand one hundred and twenty-four ten-thousandths. What is the place value of the 3? 2? 1? 4? 31? 12? 124? 312?

567·089 is read: Five hundred and sixty-seven and eighty-nine thousandths. It is a **mixed decimal** and is read as a *whole number* and a *fraction.*

Now, in integers, any part of a number is read by giving the part read the *place* name of its right-hand digit.

Thus in 567089 the part underlined is: Six hundred and seventy *hundreds.*

The same is true of decimals and 567·089 may be read: Five hundred and sixty-seven thousand and eighty-nine *thousandths.*

Give place names to the following parts of 567·089: 67, 56, 6, 8, 9, 670, 708, 6708, 7089, 89.

EXERCISE 65

1. Read the following: ·3, ·7, 1·3, 24·6, 306·8, 2001·4.

2. Read the following: ·42, ·75, ·08, 3·51, 2·04, 18·63, 27·01, 30·87, 60·03, 218·42, 500·05, 3001·08.

3. Read the following: ·825, ·416, ·307, ·006, 2·412, 3·005, 64·157, 309·043, 700·009, 6020·075.

4. Read the following: ·4165, ·0216, ·0035, ·0008, 756·3168, 4218·0032.

5. Write: Six-tenths, five *and* three-tenths, seventy-two tenths, six hundred *and* three-tenths, eighty-four hundredths, seven *and* fifty-six hundredths, nine hundredths, three hundred and four *and* five hundredths, seven thousandths, fourteen thousandths, eleven *and* eight hundred and fifty-seven thousandths, six thousand two hundred and eight thousandths, five thousand two hundred and nine *and* twenty-seven ten-thousandths.

6. Express as tenths: 5 units, 3 units, 4 units and 3 tenths.

7. Express as hundredths: 9 units, 7 units, 3 tenths, 8 tenths, 5 units and 7 tenths, 3 units 6 tenths 4 hundredths.

8. Express as thousandths: 4, ·03, 5, 3·2, 92·01, 12.

9. Write in decimal form: $\frac{7}{100}$, $\frac{15}{1000}$, $\frac{4}{1000}$, $\frac{82}{10}$, $\frac{806}{1000}$, $\frac{827}{10000}$, $16\frac{7}{10}$, $365\frac{8}{1000}$.

10. Examine: 6·3, 06·3, 6·30, 60·3 and 6·03 and state what change is made in the value of a number when a cipher is placed before the integral part; after the decimal part; between the decimal point and the first *significant* digit to the left; between the decimal point and the first *significant* digit to the right.

NOTE: The digits 1, 2, 3, 4, 5, 6, 7, 8, 9, are called *significant* digits.

ADDITION AND SUBTRACTION OF DECIMALS

In the addition of integers how are the addends arranged? In subtraction how is the subtrahend arranged under the minuend?

The same order is observed in the case of addition and subtraction of decimals. Units are placed under units, tens under tens, tenths under tenths, etc.

This arrangement will be carried out most easily by placing the decimal points under each other.

Example 1: Find the sum of 6·32, 45·817, ·09, 867·8256, 3·14.

Arrange these thus:

$$\begin{array}{r} 6{\cdot}32 \\ 45{\cdot}817 \\ {\cdot}09 \\ 867{\cdot}8256 \\ 3{\cdot}14 \\ \hline \end{array}$$

The digits in the same columns are added as if all were whole numbers, 10 units of any order being equal to 1 unit of the next higher order.

Example 2: From 365·04 take 98·736.

The work is set down thus:

$$\begin{array}{r} 365{\cdot}04 \\ 98{\cdot}736 \\ \hline 266{\cdot}304 \end{array}$$

From what has the 6 thousandths been taken?

ORAL EXERCISE

Find the value of:

1. $\cdot 3 + \cdot 4 + \cdot 6 + \cdot 8 + \cdot 9$.
2. $\cdot 5 + \cdot 24 + \cdot 26 + \cdot 123$.
3. $\cdot 6 + \cdot 8 + \cdot 04 + \cdot 005$.
4. $6{\cdot}2 + 3{\cdot}5 + 2{\cdot}81 + 3{\cdot}44$.
5. $\cdot 7 - \cdot 3$, $\cdot 64 - \cdot 25$, $81 - \cdot 07$.
6. $\cdot 53 - \cdot 4$, $7 - \cdot 8$, $6{\cdot}63 - \cdot 2$.
7. $6 - \cdot 25$, $9 - 3{\cdot}8$, $10 - 5{\cdot}14$.
8. $\cdot 23 + \cdot 46 - \cdot 17$, $\cdot 42 + \cdot 88 - \cdot 53$.
9. $8{\cdot}3 + 2{\cdot}9 - 4{\cdot}03$, $6{\cdot}1 - 3{\cdot}04 + 2{\cdot}8$.
10. \$1·35 + \$·62 − \$·48, \$3·42 − \$1·86, \$0·75 + \$0·4.

EXERCISE 66

Find the value of:

1. $\cdot 6 + \cdot 9 + \cdot 46 + \cdot 38 + \cdot 834 + \cdot 416$.
2. $9{\cdot}3 + 18{\cdot}7 + 625{\cdot}4 + 3{\cdot}8 + 53{\cdot}8$.
3. $29{\cdot}84 + 15{\cdot}83 + 26{\cdot}37 + 69{\cdot}45 + 37{\cdot}98$.
4. $34{\cdot}46 + 87{\cdot}5 + 9{\cdot}432 + 78{\cdot}6 + 6{\cdot}7$.
5. $439{\cdot}6 + 5376{\cdot}84 + 6397{\cdot}882 + 3261{\cdot}984$.
6. $3125{\cdot}642 + 64{\cdot}85 + 6{\cdot}928 + 74{\cdot}3 + 11{\cdot}42 + \cdot 8$.
7. $\cdot 284 + 3{\cdot}8 + 41 + \cdot 9 + \cdot 04 + 63 + 8{\cdot}463 + \cdot 915$.
8. \$3·90 + \$87 + \$14·04 + \$·015 + \$26 + \$9·875.
9. \$28·5 + \$37·876 + \$715 + \$0·46 + \$6·218 + \$0·75.
10. $\cdot 45 - \cdot 27$, $7{\cdot}85 - 3{\cdot}97$, $82{\cdot}4 - 57{\cdot}6$.
11. $24{\cdot}36 - 15{\cdot}4$, $54{\cdot}8 - 26{\cdot}87$, $8{\cdot}35 - 2{\cdot}463$.
12. $92 - 4{\cdot}36$, $16 - \cdot 425$, $67{\cdot}82 - 48$.
13. $23{\cdot}41 - \cdot 5178$, $7641 - 2674{\cdot}853$.
14. $49{\cdot}4 - 3{\cdot}86217$, $1003 - 5{\cdot}846$, $10 - \cdot 00423$, $937{\cdot}436 - 488{\cdot}679$.
15. $64{\cdot}25 + 37{\cdot}5 - 8{\cdot}49 + 1{\cdot}03 - 75{\cdot}4$.

16. $\cdot 413 - \cdot 284 + \cdot 3 - \cdot 06 + \cdot 98 - \cdot 5$.

17. $3664{\cdot}14 - 315{\cdot}004 + 87{\cdot}196 - 468{\cdot}82$.

18. $88{\cdot}75 - 62{\cdot}003 + 43{\cdot}9 - 1{\cdot}0009 + 635{\cdot}37 - 36$.

19. $4532{\cdot}04 - \cdot 0049 + \cdot 837 - 2146{\cdot}5 - 947{\cdot}3$.

20. $53{\cdot}8 + 8{\cdot}64 - 46{\cdot}008 - 3{\cdot}4 + 19 - \cdot 0547$.

21. $\$64{\cdot}20 + \$38{\cdot}124 - \$0{\cdot}85 - \$23{\cdot}1 + \$5 - \${\cdot}0575$.

22. From Montreal to Ottawa is 115·7 miles, from Ottawa to North Bay 244·3 miles, from North Bay to Sudbury 79·2 miles, and from Sudbury to Fort William is 556·3 miles. How far is it from Montreal to Fort William?

23. In January a householder used 3·4 thousand cubic feet of gas; in February, 4·03 thousand cubic feet; in March, 5·275 thousand cubic feet; in April, 2·17 thousand cubic feet; in May, 4·8 thousand cubic feet; and in June, 1·385 thousand cubic feet. How many thousand cubic feet did he use in the six months?

24. In the first of three farms there are 87·6 acres of land, in the second there are 14·9 acres more than in the first, and in the third there are 42·57 acres less than in the first and second together. How many acres of land are there in the three farms?

25. A steamship goes 385·4 miles the first day, 296·57 miles the second day, 347 miles the third day, 398·875 miles the fourth day, and 286·35 miles the fifth day. How far has it yet to go to complete a voyage of 2000 miles?

26. Out of 100 parts of bread, 8·9 parts are nitrogenous matter, 56·7 parts starchy, 4·1 parts fatty, and 1·1 parts salty. The rest is water. How many parts of water are there?

27. Of apple-pie, ·031 is nitrogenous, ·428 starchy, ·018 salty, and ·425 is water. The rest is fatty. How much of apple-pie is fatty?

28. A package of oatmeal weighing 3·875 pounds falls into water, and when it is taken out it weighs 4·25 pounds. How much water did it take up?

MULTIPLICATION OF DECIMALS

What is the value of $\frac{3}{10} \times 3$? of $3 \times \frac{7}{10}$? of $19 \times \frac{7}{10}$?
What is the value of $\frac{3}{10} \times \frac{9}{10}$? of $\frac{17}{10} \times \frac{8}{10}$? of $\frac{23}{10} \times \frac{13}{10}$?
What is the value of $\frac{19}{100} \times 11$? of $\frac{7}{100} \times \frac{1}{10}$? of $\frac{9}{100} \times \frac{33}{10}$?

We express these results, decimally, as follows:

$\cdot 3 \times 3 = \cdot 9$, $3 \times \cdot 7 = 2 \cdot 1$, $19 \times \cdot 7 = 13 \cdot 3$.
$\cdot 3 \times \cdot 9 = \cdot 27$, $1 \cdot 7 \times \cdot 3 = \cdot 51$, $2 \cdot 3 \times 1 \cdot 3 = 2 \cdot 99$.
$\cdot 19 \times 11 = 2 \cdot 09$, $\cdot 07 \times \cdot 1 = \cdot 007$, $\cdot 09 \times 3 \cdot 3 = \cdot 297$.

It is evident that the only difference between the multiplication of whole numbers and the multiplication of decimals is, that in the latter the decimal point has to be placed.

Compare the number of digits in the decimal part of each of the above products with the number of digits in the decimal parts of the multiplicand and multiplier.

Example 1: Multiply $37 \cdot 85 \times 4 \cdot 7$.

What is the *place* value of the 5? of the 7? What will be the *place* value of the product 35? In what place after the decimal, then, should the 5 be put? The work is done thus:

$$\begin{array}{r} 37 \cdot 85 \\ 4 \cdot 7 \\ \hline 26495 \\ 15140 \\ \hline 177 \cdot 895 \end{array}$$

Explanation: The 5 is thousandths, the 9 is hundredths, the 8 tenths and the 7 units. Hence the decimal point is placed between the 7 and 8.

EXERCISE 67

Multiply:

1. 3·7 by 8.
2. 4·59 by 9.
3. 78·643 by 67.
4. 5816 by ·45.
5. 8·3 by 7·6.
6. 9·42 by 5·1.
7. 92·77 by 34·8.
8. ·273 by 9·5.
9. ·348 by ·05.
10. 476·54 by 3·62.
11. 354 by ·726.
12. 4375 by 5·77.
13. ·819 by 6·14.
14. 12·28 by 3·9.
15. 2·43 by ·008.

16. Multiply 4·32156 by 10, by 100, by 1000. What change does each of these multipliers make in the position of the decimal point? How then could we at once write down the product for a decimal multiplied by 10? by 100? by 1000? by 10000?

17. Write the product of 5·4637 by 1000, 137·846 by 100, 29·7836 by 10000, 948·75 by 1000.

18. The length of a metre is 39·37 inches. How many inches are there in 7·5 metres?

19. How many ounces are there in ·4375 lb.?

20. How many feet are there in ·525 miles?

21. How many pounds are there in ·348 tons?

22. Reduce ·625 bu. (*a*) to pecks, (*b*) to pecks and gallons.

23. Reduce ·34 wk. to days, hours, minutes, etc.

24. The circumference of a wheel is 3·1416 times its diameter. What is the circumference of a wheel 6 ft. in diameter?

25. A merchant sells his goods at a gain of ·125 of their cost. What will he receive for goods which cost him $480?

26. One pound of milk will make ·1087 lb. of cheese. How many pounds of cheese can be made from 360 lb. of milk?

27. What will be the cost of 80·4 tons of coal at $6·85 a ton?

28. A ball team won ·625 of all games played. If the number of games played was 88, how many games did the team win?

29. In a certain school there were enrolled during the month of January 40 pupils. The average daily attendance was ·875 of the number on the roll. What was the average daily attendance?

30 If it cost 13·2 cents a sq. ft. to build a cement walk, what will be the cost of a walk 80 ft. long and 4·5 ft. wide?

DIVISION OF DECIMALS

In the division of integers by integers, how is the first digit in the quotient obtained? How may its place value be known?

If 65879 be divided by 53, what is the first digit in the quotient? What is its place value? What then must be the place value of the second digit in the quotient? of the third? Again if 658·79 be divided by 53, what will the first digit in the quotient be? What is its place value? What is the place value of the second digit? of the third? of the fourth?

658·79 may, therefore, be divided by 53 as follows:

```
   12·43
53)658·79
   53
   ---
   128
   106
   ---
    227
    212
    ---
     159
     159
     ---
```

If the divisor had been 5·3 or ·53, how could it be changed to the integer 53? What change must then be made in the dividend to leave the quotient unchanged?

It is clear, therefore, that in the division of decimals the divisor can always be made a whole number. For instance, $16·37 \div 4·2 = 163·7 \div 42$ and $·00378 \div ·64 = ·378 \div 64$. Hence the division of decimals can always be performed as in the above example. It will be seen that in dividing by a whole number the decimal point in the quotient comes immediately above that in the dividend.

EXERCISE 68

Divide:

1. 87·5 by 35.
2. 32·68 by 19.
3. ·1518 by 46.
4. ·7332 by 6.
5. 2·64 by ·8.
6. 3·75 by ·25.
7. ·9 by ·12.
8. 9·6188 by 3·46.
9. 30·5 by 61.
10. 1131·264 by 4·8.
11. 3·381 by ·00147.
12. 22·104 by ·0072.
13. 8·31183 by 23·05.
14. ·35209 by 25·7.
15. 24 by ·192.
16. 640 by ·256.
17. 5658 by ·0123.
18. ·004 by 2·5.
19. ·12 by ·0006.
20. 59·4204 by 5860.
21. ·461071 by 122·3.
22. ·348336 by ·492.
23. 5 by 125.
24. 8 by 625.

25. Divide 437·25 by 10, by 100, by 1000, by 10000. What change is made in the position of the decimal point by each of these divisions?

26. What is the quotient when 4368 is divided by 100? When 37·5 is divided by 10000?

27. Divide 4 by 5, 3 by 60, 8 by 500, 7 by 8, 1 by 16, 2 by 25, 9 by 20.

Divide, correct to four places of decimals:

28. 64·375 by 9·573.
29. 4·50775 by 123·5.
30. 6 by 17.
31. 3 by 7.
32. 1 by 13.
33. 14 by 1·9.

34. If a pound of sugar costs 4·25c., how many pounds can be bought for 85c.?

35. How much wheat at $·95 a bushel must a man sell to pay a debt of $455·05?

36. A train runs a distance of 36·8 miles in 1·15 hours. What is its rate in miles an hour?

37. What is the width of a sheet of paper whose length is 12·5 in. and which contains 92·5 sq. in.?

38. How many hundredweight of flour are there in 3620 lb.? What will the flour be worth at $2·65 a hundredweight?

39. Find the value of 4350 board feet of lumber at $45·50 a thousand feet.

40. Find the value of 600 cu. ft. of gas at $1·35 a thousand cubic feet.

41. In an Imperial gallon there are 277·274 cu. in. Find to two decimal places the number of gallons of water which will be required to fill a rectangular trough 3 ft. wide, 4 ft. deep, and 9 ft. long.

42. The total attendance at a certain school for the month of March, in which there were 23 teaching days, was 652. Find to two places of decimals the average daily attendance.

43. A base-ball team played 78 games of which it won 53. Find to three places of decimals its winning average.

44. The Cook Cheese Manufacturing Company during the season of 1909 used 614095 lb. of milk in making cheese which was sold for $6768·30. If the season's expenses amounted to $1096·54, find, in cents, to 6 places of decimals, the price which they could pay a pound for the milk.

45. If, in Question 44, the amount of cheese manufactured was 59473 lb., find to two decimal places the number of pounds of milk required to make one pound of cheese.

46. Find the amount paid to patrons who supplied 78040 lb. and 65457 lb. of milk.

REDUCTION OF DECIMALS TO VULGAR FRACTIONS

Example: Express ·32 as a vulgar fraction:

$$\cdot 32 = 32 \text{ hundredths} = \tfrac{32}{100} = \tfrac{8}{25};$$

again $6\cdot 32 = \tfrac{632}{100} = 6\tfrac{32}{100} = 6\tfrac{8}{25}$.

EXERCISE 69

Express as vulgar fractions in their lowest terms:

1. ·3, ·6, ·8, ·37, ·45, ·64.
2. ·05, ·025, ·001, ·675, ·317, ·144.
3. ·7375, ·0112, ·0175, ·0674, ·3225.
4. 4·75, 10·25, 12·064, 7·275, 15·12.

REDUCTION OF VULGAR FRACTIONS TO DECIMALS

Regarding a vulgar fraction as the quotient obtained by dividing the numerator by the denominator, $\frac{3}{4} = 3 \div 4$. (See page 105.)

To express $\frac{3}{4}$ as a decimal divide 3·000 by 4 and the result is ·75.

Example: Reduce $\frac{3}{7}$ to 4 places of decimals:

$$\frac{3}{7} = 3 \div 7 = ·4285.$$

EXERCISE 70

1. Doing the work mentally, reduce to decimals:
$\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{10}$, $\frac{3}{10}$, $\frac{7}{10}$, $\frac{9}{10}$, $\frac{1}{20}$, $\frac{3}{20}$, $\frac{7}{20}$, $\frac{9}{20}$, $\frac{1}{25}$, $\frac{2}{25}$, $\frac{3}{25}$, $\frac{4}{25}$, $\frac{6}{25}$, $\frac{7}{25}$, $\frac{8}{25}$, $\frac{9}{25}$.
2. Reduce to decimals:
$\frac{3}{16}$, $\frac{7}{16}$, $\frac{17}{25}$, $\frac{9}{32}$, $\frac{15}{64}$, $\frac{23}{25}$, $\frac{69}{125}$, $\frac{7}{625}$, $\frac{17}{40}$, $\frac{39}{80}$, $\frac{57}{250}$.
3. Reduce to four places of decimals:
$\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{11}$, $\frac{8}{13}$, $\frac{7}{15}$, $\frac{14}{17}$, $\frac{11}{18}$, $\frac{16}{23}$, $\frac{9}{14}$, $\frac{61}{112}$.
4. Find to four places of decimals the value of:
$$\frac{5}{12} + ·173 - \frac{1}{7} + ·9624 - \frac{16}{29}.$$
5. Find to the nearest cent the value of:
$$\$\frac{3}{5} + \$·55 + \$\frac{3}{8} - \$1·38 + \$\frac{5}{11}.$$
6. A man who owns ·625 of a stock of goods sells $\frac{3}{4}$ of his share. What decimal of the stock has he left? What will be the value of what he now owns if the whole stock is worth \$6400?

7. If a work-day is 12 hr., what fraction of a work-day is 10 hr. 30 min.? To what decimal is this fraction equal? At \$2·40 a day of 12 hr. how much would a man earn in 10 hr. 30 min.?

8. What decimal of 1 bu. is 2 pk. 3 qt. 1 pt.? What will 2 pk. 3 qt. 1 pt. of chestnuts be worth at \$3·20 a bushel?

9. What decimal of 11 lb. 9 oz. is 2 lb. 5 oz.? If 11 lb. 9 oz. of butter be worth \$3·70, what will be the value of 2 lb. 5 oz.?

10. Express $2\frac{1}{2}$ qt. as a decimal of 1 gal. What will 3 gal. $2\frac{1}{2}$ qt. of wine cost at \$4 a gal.?

EXERCISE 71

1. A cubic foot of water weighs 10 lb. If cast-iron is 7·2 times as heavy as water, how many cubic feet of cast-iron will weigh as much as 3060 cu. ft. of water?

2. Find the number of pounds in ·175 of a ton + ·235 of a hundredweight + ·35 of a pound.

3. Add together ·029 of 1 da. 3 hr., ·45 of 11 hr. 10 min., and ·89 of an hour, and then express the answer in minutes and the decimal of a minute.

4. Express, in inches, the difference between $\frac{3}{8}$ yd. and ·875 ft.

5. A base-ball player's "batting average" is calculated by finding what decimal the number of his "safe hits" is of the number of times he was "at bat." Find the batting average, to three decimal places, of the player who out of 59 times "at bat" made 18 "safe hits."

6. A cricketer made 1148 runs in 68 innings. Calculate to 2 decimal places the average number of runs an innings.

7. Find to two decimal places the average weight of 7 boys whose individual weights were: 88 lb., 95 lb., 93 lb., 110 lb., 102 lb., 87 lb., and 101 lb.

8. Divide \$35·36 among 4 men and 3 boys giving to each boy ·14 of a man's share.

9. Find the cost of 6250 laths at 32 cents a hundred.

10. Out of 100 parts of rice ·3 parts are fat, ·4 parts are ash, 7·8 parts are proteid, and 79·2 parts are starch. The rest is water. How much water will there be in 60 lb. of rice?

11. At a certain mine a ton of iron ore yields ·55 tons of pure iron. How much pure iron will there be in 86·4 tons of ore?

12. A gentleman finds that he spends ·15 of his salary for insurance, ·48 for living expenses and ·10 for charity. He invests the remainder, $810. Find his total salary.

13. A railway time-table shows Kenora 1289·4 miles and Winnipeg 1414·9 miles west of Montreal. An express leaves Kenora at 5.30 a.m. and arrives at Winnipeg at 9.45 a.m. Find the average rate of the express in miles an hour.

14. Find the value of: $\cdot3 \times \cdot03 - \cdot01 \times \cdot002 \times 10\cdot4$.

15. By cancellation, find the value of:

$11\cdot52 \times 1\cdot04 \times 3\cdot36$ divided by $\cdot48 \times 2\cdot6 \times 76\cdot8$.

16. By cancellation, find the value of:

$6\cdot6 \times 2\cdot24 \times 1\cdot56 \times 1\cdot08$ divided by $3 \times \cdot44 \times 2\cdot8 \times \cdot81$.

17. A farmer sows 2·5 bu. of oats an acre on 10·25 acres which yielded a crop averaging 38·4 bu. an acre. If his seed grain cost him $·56 a bu. and his other expenses amounted to $3·60 an acre, how much will he gain by selling his oats at $·48 a bu.?

18. The Maple Leaf Cheese Factory in 1909 received from its patrons 2,259,294 lb. of milk. The receipts from sale of cheese were $23874·95 and the expenses were $2381·92. The remainder of the receipts was paid to the patrons for milk supplied. Find to five decimal places the price paid for the milk in cents per pound. What would be paid to patrons supplying 90934 lb., and 68934 lb. of milk?

19. If in Question 18 the total cheese manufactured was 207746 lb., find to two decimal places the number of pounds of milk required to make 1 lb. of cheese.

THE METRIC SYSTEM OF WEIGHTS AND MEASURES

In almost every civilized country, with the exception of Great Britain, her Colonies, and the United States, the **metric system** of Weights and Measures is in general use.

In this system there is *one standard unit*, that of length, called a **metre**, from which are obtained the units of area, of volume, and of weight. It is the one ten-millionth part of the distance from the equator to the pole.

Prefixes: In learning the Metric Tables it is necessary to memorize carefully the following prefixes and their meanings:

Deka	= 10.	Deci	= $\frac{1}{10}$.
Hecto	= 100.	Centi	= $\frac{1}{100}$.
Kilo	= 1000.	Milli	= $\frac{1}{1000}$.

MEASURES OF LENGTH

The unit of length is the *metre*, equal to 39·37 inches.

10 **millimetres**	(mm)	. . . . .	1 centimetre
10 **centimetres**	(cm)	. . . . .	1 decimetre
10 decimetres	(dm)	. . . . .	1 **metre**
10 **metres**	(m)	. . . . .	1 dekametre
10 dekametres	(Dm)	. . . . .	1 hectometre
10 hectometres	(Hm)	. . . . .	1 **kilometre** (Km)

NOTE: The most important units are in heavy type.

Approximately, 8 kilometres = 5 miles.

Example 1: How many millimetres are there in 43 dekametres?

	Dm	m	dm	cm	mm	
43 dekametres =	43	0	0	0	0	= 430000 millimetres.

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Explanation: Dekametres are the units of the fourth denomination *above* millimetres, and the value of each denomination is ten times the value of the denomination next *below* it. Therefore, the number of millimetres is *10000 times* the number of dekametres.

Example 2: How many metres are there in 564 centimetres?

m dm cm
564 centimetres = 5 6 4 = 5·64 metres.

Explanation: Centimetres are the units of the second denomination below metres. Hence the number of metres is found by *dividing* the number of centimetres by 100.

NOTE: It is clear that in the Metric System, Reduction is merely a question of moving the decimal point.

EXERCISE 72

1. Using a metre stick, measure the length of objects about you: Your desk, the window, the school-room floor, the doors, the walls, the black-board, etc.

2. How many metres in a kilometre? in a hectometre? in a centimetre? in 5 dekametres? in 3·2 kilometres? in 1479 centimetres?

3. Express 3142·69 metres as hectometres, as kilometres, as millimetres.

4. Read 2415·637 metres, giving the *metric* denomination of each digit.

5. Express 457·382 decimetres as centimetres, as hectometres, as millimetres, as kilometres.

6. What is the difference in inches between 4 yards and 4 metres?

7. What will 6 metres of cloth cost @ 85c. a metre?

8. What will be the cost of 3·85 metres of ribbon @ 40c. a metre?

9. The distance between two towns is 20 Km. What is the distance in miles? If the distance is 20 mi., what is the distance in kilometres?

10. Find the value in metres of:

$$4{\cdot}2 \text{ Km} + 3{\cdot}75 \text{ Hm} + 9{\cdot}436 \text{ m} + 42{\cdot}5 \text{ cm}.$$

11. Find the value in metres of:

$$23{\cdot}4 \text{ dm} \times 6 - 348{\cdot}2 \text{ dm} \div 4 + 621 \text{ m}.$$

12. If a man's step averages 7 dm in length, how many steps will he take in going 3·5 Km?

SURFACE OR SQUARE MEASURE

100 square millimetres	1 sq. centimetre
100 square centimetres	1 sq. decimetre
100 square decimetres	1 sq. metre
100 square metres	1 sq. dekametre
100 square dekametres	1 sq. hectometre
100 square hectometres	1 sq. kilometre

EXERCISE 73

1. Reduce 346 sq. mm to sq. Hm, to sq. cm.
2. Reduce 4683 sq. Dm to sq. dm, to sq. Hm.

CUBIC MEASURE

1000 cubic millimetres	1 cubic centimetre
1000 cubic centimetres	1 cubic decimetre
1000 cubic decimetres	1 cubic metre

EXERCISE 74

1. Express 6·214 cu. m in cubic decimetres, in cubic centimetres.

2. Express 3825 cu. cm in cubic decimetres, in cubic metres.

3. How many bricks 2·1 dm long, 1·1 dm wide, and 6 cm thick will it require for a solid brick wall, without mortar, 63 m long, ·48 m wide, and 2·2 m high?

4. How many cubic metres of earth will be removed to make a rectangular cellar 1·2 Dm long, 8 m wide, and 2 m high?

5. How many cubic metres in your school-room?

MEASURES OF CAPACITY

The unit of capacity used for measuring liquids and dry substances is the *litre*. It equals a cubic decimetre.

10 millilitres (ml)	1 centilitre
10 centilitres (cl)	1 decilitre
10 decilitres (dl)	1 **litre**
10 litres (l)	1 dekalitre
10 dekalitres (Dl)	1 hectolitre
10 hectolitres (Hl)	1 **kilolitre** (Kl)

EXERCISE 75

1. Construct out of cardboard a cubic box, the length of one side being 1 decimetre.

2. Read 364·75 l, giving the metric denomination of each digit.

3. How many decilitres in 2 dekalitres ? in 4 millilitres ?

4. How many litres will it take to fill a rectangular tank 3 m long, 2 m wide, and 1·5 m deep ?

5. What will be the value of 6·34 kilolitres of wine at 50c. per litre ?

6. A box measuring 30 cu. cm will contain how many litres ?

MEASURES OF WEIGHT

The unit of weight is the *gramme*. The gramme is the weight of a cubic centimetre of pure water at its greatest density.

10 milligrammes (mg)	1 centigramme
10 centigrammes (cg)	1 decigramme
10 decigrammes (dg)	1 **gramme**
10 grammes (g)	1 dekagramme
10 dekagrammes (Dg)	1 hectogramme
10 hectogrammes (Hg)	1 **kilogramme** (Kg)

A kilogramme (sometimes written kilo) is about $2\frac{1}{5}$ lb. A metric ton (T) is 1000 kilogrammes or nearly 2204·6 lb.

EXERCISE 76

1. Express as grammes:
123 cg, 2980 Kg, 4900 mg.

2. Express as milligrammes:
25 dg, 32 g, 124 Kg.

3. Express as kilogrammes:
64000 dg, 425 T, 863 Hg.

4. What will be the weight in kilogrammes of a rectangular mass of water 1 m in length, 5 dm in width and 2 dm in height?

5. Lead is 11·3 times as heavy as water. What will be the weight in kilogrammes of a rectangular bar of lead ·52 m long, ·35 m wide, and ·125 m thick? How many pounds will it weigh?

6. Find as a decimal of a gramme the value of:
·06325 Kg + ·3975 g + ·91 mg.

7. Find the value of 32·6 Kg of copper at 2·5 francs per kilogramme.

EXERCISE 77

1. What is the difference in feet between 16 Km and 10 mi?

2. A train runs at the rate of 48 Km an hour. In how many minutes will it go 660 m?

3. Find the cost of 80 Kg 125 g of butter at 64c. per kilogramme.

4. Gold is 19·3 times as heavy as water. Find the weight in kilogrammes of a solid gold cube whose edge is 5 cm.

5. How many cu. m of wood are there in a pile 6·40 m long, 1·50 m wide, and 2 m high?

6. How many litres of grain will fill a bin 1·25 m high, 1·20 m wide, and 3 m long?

7. The distance from Toronto to Montreal is about 338·5 mi. What is the distance in kilometres?

8. How many metres of carpet 30 cm wide will it require for a floor 7·5 m long and 6 m wide?

COMMERCIAL ARITHMETIC

Substitute the proper numerators in the following:

$\frac{3}{4} = \frac{\quad}{8} = \frac{\quad}{12} = \frac{\quad}{84} = \frac{\quad}{27} = \frac{\quad}{100}$.

$\frac{2}{7} = \frac{\quad}{21} = \frac{\quad}{49} = \frac{\quad}{15} = \frac{\quad}{80} = \frac{\quad}{100}$.

$\frac{4}{5} = \frac{\quad}{100}$, $\frac{5}{8} = \frac{\quad}{100}$, $\frac{2}{3} = \frac{\quad}{100}$.

The fraction $\frac{1}{100}$ is found to be so useful in business transactions that a special name and symbol are given to it. The name is **per cent.** and the symbol is %. For instance, $\frac{5}{100} = 5\%$, or 5 per cent.; 3 per cent. $= 3\% = \frac{3}{100} = .03$.

$$2\tfrac{1}{2}\% = \frac{2\frac{1}{2}}{100} = .025; \qquad 35\% = \frac{35}{100} = .35;$$

$$\tfrac{6}{8} = \frac{60}{100} = .60 = 60\%; \qquad .175 = \frac{17\frac{1}{2}}{100} = 17\tfrac{1}{2}\%.$$

EXERCISE 78

1. What is the per cent. equivalent for $\frac{7}{10}$, $\frac{9}{20}$, $\frac{4}{5}$, $\frac{16}{25}$, $\frac{427}{500}$?

2. Find the per cent. equivalents for $\frac{1}{6}$, $\frac{5}{8}$, $\frac{2}{3}$, $\frac{7}{30}$, $\frac{8}{7}$, $\frac{17}{4}$.

3. It will be found useful to remember the per cent. equivalents for the eighths, the sixths, the tenths, the twentieths, and the twenty-fifths. Make out the table for these.

4. Find the per cent. equivalents for: .07, .0375, .125, .0025, 3.5, 14.25.

5. Find the vulgar fraction equivalents, and the decimal fraction equivalents for:

$$6\%,\ 3\tfrac{1}{4}\%,\ 17\tfrac{1}{2}\%,\ 125\tfrac{5}{8}\%,\ 8\tfrac{1}{3}\%.$$

6. Find: 12% of 375, that is, $\frac{12}{100}$ of 375; 28% of 225; $6\frac{1}{4}\%$ of 369; $9\frac{1}{11}\%$ of \$4323; $62\frac{1}{2}\%$ of 5424 acres.

7. What fraction is 3 yd. of $16\frac{2}{3}$ ft.? what per cent.?

8. What per cent. is: 8 of 48? 36 of 96? \$42 of \$630? 24 oz. of 2 lb.? $5\frac{1}{3}$ qt. of 3 pk.?

9. What per cent. is 16 of 32? \$1·50 of \$45? 35 of $3\frac{1}{2}$ doz.? $6\frac{3}{4}$ of $4\frac{1}{2}$? 81 ft. of 7 ft. 6 in.?

10. Find the number of which 24 is 3%, 63 is $4\frac{1}{2}$%, 728 is 104%, 6 is $\frac{1}{5}$%, 32 is $\frac{2}{3}$%, 935 is 85%.

11. A merchant sells goods valued at \$450, but is able to collect only 80% of the amount. Find the amount collected.

12. At a certain examination the total marks given were 4200. How many marks did a student receive who got $65\frac{1}{2}$% of the total?

13. The population of a town is found to have increased 20% every 10 years. Its population in 1881 was 6250. What was its population in 1891? in 1901?

14. The average daily attendance at a school for the month of October was 92% of the number on the roll. If the number on the roll was 325, find the average attendance.

15. A casting which weighs 480 lb. contains $96\frac{1}{2}$% of copper, and the rest is tin. How many pounds of tin are there?

16. A man receiving a salary of \$1200 a year spends 14% of it for board, 10% of it for clothing, and $13\frac{1}{2}$% of it for other expenses. How much can he save in 6 years?

17. A library contains 4800 volumes, of which 1440 are fiction. What fraction of the library is fiction? What per cent.?

18. A banker lends a farmer \$469 and receives \$18·76 for the use of it. What per cent. is this of the sum loaned?

19. Of 8280 candidates who wrote on a certain examination 3925 passed. What per cent. failed?

20. To 63 gal. of wine there are added 12 gal. of water. What per cent. of the mixture is the water?

21. Of 51,000,000 bu. of wheat sent out of Canada all but 850,000 was sent to Great Britain. What per cent. of the wheat exported did Great Britain receive?

22. An article which cost \$3·60 was sold for \$4·20. What fraction of the cost was the gain? What per cent.?

NOTE: The gain or loss is usually expressed as a per cent. of the cost.

23. I buy a horse for \$150 and sell it so as to gain \$30. For what per cent. of the cost do I sell it?

24. The cost of a farm was \$4500, and the selling price was \$4960. What was the gain per cent.?

25. By selling a carriage for \$138 the seller makes a profit of 15%. What does it cost him? At what should he sell it to make $33\frac{1}{3}\%$? What fraction of the cost is the selling price in each case?

26. Goods damaged by fire were sold at a loss of 16%. If the total loss was \$3600, find the original cost of the goods.

27. A grocer sold oranges for \$16·50, gaining thereby 10%. What would have been his loss per cent. had he sold them for \$12·75?

28. A tea merchant mixes 40 lb. of tea at 45c. per lb. with 20 lb. at 30c. per lb. What is his gain per cent. by selling the mixture at 50c. per lb.?

29. If $16\frac{2}{3}\%$ is gained by selling a sewing-machine for \$42, for what should it be sold to gain $33\frac{1}{3}\%$?

30. A sold goods to B at a profit of 10%, and B sells them to C at a profit of 15%. If C paid \$506 for the goods, what did they cost A?

COMMISSION

One person sometimes employs another to transact for him certain business, such as buying, selling, or renting property, buying or selling goods, collecting accounts, soliciting orders, etc. The person so employed is called an **agent,** a **broker,** or a **commission merchant,** according to the nature of the business in which he is engaged.

The person employing the agent is called the agent's **principal.**

Name some agents in your neighbourhood, and state the nature of the business in which each is engaged.

Instead of receiving a wage or salary, these agents usually receive a **commission,** that is, **a certain per cent. of the value of the business they do.** What advantage is there in this method ?

EXERCISE 79

1. At 3% what is an agent's commission for selling a farm for $4000 ?

2. At $2\frac{1}{2}$% what is an agent's commission for buying a house for $2400 ?

3. For collecting accounts an agent receives a commission of 5%. What will be the amount of his commission for collecting $750 ?

4. A book agent receives 25% on his sales. What will be his commission for selling 6 sets of books at $12·50 a set ?

5. An agent sells 450 bbl. of apples at $4 a bbl. on a commission of $3\frac{1}{4}$%. How much will his principal receive from the sales ?

NOTE: It is customary to deduct the commission for selling, from the amount of the sales and transmit the balance to the principal. The amount returned to the principal is called the **net proceeds.**

6. How much shall I have to send my agent in order that he may purchase for me 600 bu. of oats, at 35c. a bushel, after deducting his commission of $4\frac{1}{2}$% ?

7. For renting a house a real estate agent charges 2% of the annual rent. What will be his commission for renting a house at $18 a month ?

8. For selling $6200 worth of goods an agent receives a commission of $155. What was the rate charged ?

9. A commission merchant bought goods worth $3650 and received $54·75 as his commission. What was the rate charged?

10. A dealer sent his agent $1842·75, instructing him to retain his commission and purchase goods with the balance. The agent's commission was $22·75. What was the rate charged?

NOTE: It is customary to deduct the commission for buying, from the money received, and invest the balance.

11. For selling goods on a commission of $3\frac{1}{2}\%$ an agent received $39·90. For what amount did the goods sell? How much did the agent's principal realize from the sale?

12. A broker received $24·80 for purchasing goods on a commission of $2\frac{2}{3}\%$. What was the value of the goods bought? How much did they cost the broker's employer?

13. A canvasser for a newspaper is allowed $22\frac{1}{2}\%$ of all subscriptions received by him. How many subscriptions at $4 each must he secure in order to earn $135 commission?

14. A machine agent sold twelve binders on a commission of 15%. If his employer received, as net proceeds, the sum of $1632, what was the selling price of each binder?

15. An agent remits to his principal $2488·50 as the net proceeds from the sale of 2800 bu. of wheat. Find the agent's commission if he charges at the rate of $1\frac{1}{4}\%$. Find also the selling price of the wheat a bushel.

16. A broker bought 20000 bu. of corn at 58c. a bushel, and sold it next day at 60c. Find his total commission if he charges $\frac{1}{2}\%$ for buying and also $\frac{1}{2}\%$ for selling. Find also his employer's net gain.

17. An agent sold 782 cwt. of flour on a commission of $2\frac{1}{2}\%$. If he paid $97·75 for freight and remitted his employer $2189·60, find the selling price of the flour.

18. I send my broker $2790 with which to buy cotton at 9c. a yard after retaining his commission of $3\frac{1}{3}\%$. How

many yards of cotton did he buy? What was his commission? What fraction of the cost of the cotton was the money sent?

19. A commercial traveller was offered (1) a commission of $7\frac{1}{2}\%$ on his sales or (2) a salary of \$30 a week with a 3% commission on his sales. He accepted the first offer which he found to be \$1140 a year better than the second. What amount of goods did he sell?

20. A man having bought 3000 lb. of tea instructs his broker to sell it at 51c. a pound and invest the net proceeds in sugar at 5c. a pound. How many pounds of sugar were bought if the broker's commission for buying and for selling was 2%. What was the broker's total commission?

TAXES

Who owns your school-house? What did it cost? Who furnished the money? How was it collected? Who decides how this money is to be expended? What is the money called?

In every city, etc., it is usual to appoint officers, called assessors, whose duty it is to value all the taxable property in the city, such as land, buildings, incomes, money, stock, furniture. The total of these values is called the **assessed** value, or the **assessment,** of the city, etc. The amount for which each individual is assessed is made known to him by a notice from the assessor. Try to obtain one of these notices.

The total taxes are then levied on the total assessment, and each ratepayer pays a share which is the same fraction of the total tax that his assessment is of the total assessment. The amount to be paid is calculated, usually, at the rate of a certain number of **mills on each dollar of assessment.** This rate is called the **rate of taxation.**

Each ratepayer receives a tax-bill in some such form as this:

No. on Roll.....

.....*1910*

Mr.....*Con*.....*Lot*.....

To THE MUNICIPAL CORPORATION OF NORTH MONAGHAN, Dr.

To amount of Taxes levied for 1910

On.....*acres as follows:*

TOTAL ASSESSED VALUE, $.....	.....
1.—For County Purposes, at.. .....3 Mills $.....	....
2.—For Township Purposes, at.....1⅘ Mills	...
3.—For Public School Rate, Section No. 1, at.....2 Mills	....
4.—For Public School Rate, Section No. 2, at.....2 Mills	....
5.—For Public School Rate, Section No. 3, at.....1 Mill	....
6.—For Public School Rate, Section No. 4, at.....2 Mills	....
7.—For Special Rate, Drain Tax.....	...
8.—For Statute Labour Commutation.....days at 75c. per day	....
9.—For Dog Tax	....
10.—For Arrears of Taxes.....	....
11.—For General Pub. School Rate (Co. and Tp.)..... 2 Mills	....
12.—For Separate School Rate, at.....1½ Mills	....
TOTAL TAXES	....

RECEIVED PAYMENT

.....*Collector*

EXERCISE 80

1. What is the rate of taxation when $9 taxes are paid on an assessment of $4500?

Example: $4500 pays 9000 mills
1 " 2 "

therefore rate = 2 mills on the dollar.

2. What is the rate of taxation when the taxes and the assessment are respectively: $19·80 and $3600? $178·20 and $21600? $6875 and $550,000? $141,750 and $7,875,000.

3. What amount of taxes can be raised when the assessment, and the rate on the dollar are, respectively, \$2500 and 6 mills? \$3560 and 7 mills? \$4800 and $12\frac{1}{2}$ mills? \$8822218 and 20 mills? \$166880 and 19 mills?

4. What is the assessment when the rate on the dollar, and the taxes are, respectively, 4 mills and \$1640? $7\frac{1}{2}$ mills and \$1125? 5 mills and \$13750? 16 mills and \$146000? $20\frac{1}{4}$ mills and \$982125?

5. A man has an income of \$1800. If \$600 is exempt from taxation, what amount of taxes will he be required to pay at the rate of 15 mills on the dollar? What will be his net income?

6. A municipality decides to build a bridge to cost \$15000. If the assessment of the municipality be \$10000000, what will a man who is assessed for \$4000 have to pay toward the cost of the bridge?

7. A school section is assessed for \$280000. What amount will a ratepayer, who is assessed for \$5200, have to contribute toward a teacher's salary of \$700?

8. Mr. Henderson bought a house for \$7500 on which he pays \$105 taxes. If the rate of taxation be $17\frac{1}{2}$ mills on the dollar, for what per cent. of the cost of the house is he assessed?

9. A city assessed for \$12000000 levies a rate of $13\frac{3}{4}$ mills. What amount of taxes will the city thus obtain after paying $2\frac{1}{2}\%$ for collection?

10. An Ontario city in 1909 had an assessment on real property of \$7769128, a business assessment of \$659210, and an income assessment of \$210180. Find to the nearest cent the amount of money collected for school purposes if the total school rate was 8·9 mills on the dollar.

11. A township is assessed for \$630000. The taxes are $3\frac{1}{2}$ mills for county purposes, $2\frac{4}{5}$ mills for township purposes, $2\frac{1}{2}$ mills for school purposes, $1\frac{2}{5}$ mills for other purposes, and a poll-tax of \$1·50. If 120 persons pay the poll-tax, find the total taxes raised.

NOTE: A poll-tax is a tax of a specified sum on the individual.

11. Examine the following Tax Table for a rate of $5\frac{1}{4}$ mills on the dollar and use it to find the tax paid on property assessed for \$7365.

TAX TABLE FOR RATE OF 5·25 MILLS

Assessment	Tax	Assessment	Tax	Assessment	Tax
\$1	\$0·00525	\$4	\$0·02100	\$7	\$0·03675
2	0·01050	5	0·02625	8	0·04200
3	0·01575	6	0·03150	9	0·04725

The tax on \$7000 or \$7 × 1000 = \$·03675 × 1000 = \$36·75
" " " 300 or \$3 × 100 = ·01575 × 100 = 1·575
" " " 60 or \$6 × 10 = ·03150 × 10 = ·315
" " " 5 = ·026
\$7365 = \$38·67

12. Using the above table, find the tax at $5\frac{1}{4}$ mills on \$5825. On \$690.

13. Make out tax tables like the above for rates of 4 mills, $3\frac{1}{2}$ mills and $8\frac{1}{4}$ mills.

14. Using the tax tables, find the tax on \$950 at 4 mills, \$6275 at $3\frac{1}{2}$ mills, and \$9725 at $8\frac{1}{4}$ mills.

DUTIES OR CUSTOMS

The preceding section explained and dealt with taxes levied for local purposes. There are, however, other taxes levied for the support of our federal and provincial governments.

Name some of the purposes for which these governments use their revenue. Who usually collects the money for this revenue? Where is it collected? Where is your nearest *port of entry?*

These taxes are called **duties** or **customs** and are of three classes: those levied on commodities (1) brought into

the country, (2) sent out of the country, and (3) produced or manufactured in the country.

The first are called **import duties,** the second **export duties,** and the third **excise duties.**

Name some commodities liable for each of these duties, and state what the duty in each case is.

For the purpose of duty, goods or commodities may be divided into four classes:

(1) Those which are admitted free.

(2) Those which are subject to an **ad valorem duty,** that is, a certain per cent. of their *invoiced value.*

(3) Those subject to a **specific duty,** that is, a *certain amount a pound, a yard, a piece,* etc., without reference to the value.

(4) Those subject to both an *ad valorem* and a *specific* duty. Name some goods in each of these classes.

At present goods manufactured in Great Britain and in some British colonies receive in the matter of duty, when imported into Canada, a preference over the goods imported from other countries.

A list of goods with their duties is called a **tariff.**

EXERCISE 81

Find the duty on the following:

1. 250 bbl. of apples, the duty being 40c. a bbl.

2. 625 lb. of molasses, the duty being 50c. a 100 lb.

3. $375 worth of silk, the duty being 30%.

4. 6 carriages invoiced at $110 each, the duty being 35%.

5. 560 lb. steel rivets invoiced at 6c. a lb., the duty being 75c. a 100 lb. and 25% ad valorem.

6. 80 gal. of japan invoiced at $1·15 a gal., the duty being 20c. a gal. and 22½% ad valorem.

7. A jeweller imports 2 doz. watches invoiced at $7·50 a watch. The duty being 30%, find the amount paid. Find also the total cost of the watches to the jeweller.

8. Knitted goods manufactured in England are subject to a duty of 22½%, those manufactured in the United States to 35%. How much will be saved by importing $320 worth of such goods from Great Britain instead of from the United States?

9. Dress goods imported from Great Britain are subject to 40% less duty than those imported from other countries. What will be saved by importing from Britain dress goods valued at $1600 if the regular duty imposed on such goods from other countries is 25%?

10. Select your own rates and find the duty on the following: 1 doz. axes invoiced at $1 each, 24 blankets at $3·50 a pair, 12 pair rubber boots at $2·50 a pair, 20 gal. coal oil at 18c. a gal., 8 bbl. of flour at $6·25 a bbl.

11. Find the duty on an importation of 6 tons of sugar invoiced at $3·50 a 100 lb., the duty being 31½c. a 100 lb. What will be the total cost to the purchaser?

12. An implement dealer imports the following: 6 ploughs invoiced at $20 each, 3 binders at $125 each, 4 cultivators at $40 each, 8 wagons at $75 each:

(*a*) Find the amount of duty paid, if on ploughs and cultivators there is a duty of 20%, on binders 17½%, and on wagons 25%.

(*b*) For how much must the whole be sold to yield a profit of 12½%?

13. A grocer imports 60 boxes of raisins containing 25 lb. each invoiced at 8c. a lb. He pays a specific duty of ⅔c. a lb., $7·50 for freight, and $2·50 for warehouse charges. If he retails the raisins at 3 lb. for 35c., how much does he gain? What is his gain per cent.?

14. A bookseller imports a set of books invoiced at $12 and pays a duty of 25%. If he sells the books for $18 what will be his gain per cent.?

INTEREST

Just as one who lives in a house, or on a farm, which is not his own has to pay for the use of that house or farm, so a person who borrows money has usually to pay for the use of that money.

What is the name given to the money paid for the use of houses, farms, etc.?

Money paid for the use of borrowed money is called **interest.** The money borrowed is called the **principal.**

The total sum paid to the person or persons from whom the money is borrowed is called the **amount**. Of what two sums is the amount made up?

The interest for one year is a certain per cent. of the principal. This per cent. is called the **rate of interest per annum.**

When one buys goods on credit he is, in reality, borrowing money, and is, therefore, sometimes charged interest on the value of the goods bought. So too, a banker with whom money is deposited uses borrowed money and usually pays interest on it.

It is the custom for the borrower to give to the party from whom he borrows a written acknowledgment of his debt as well as a promise to repay the money. This written document is called a **promissory note,** the form of which is as follows:

$250·00. Ottawa, 1st Nov., 1909.

Three months after date I promise to pay James H. Broadbent, or order, Two Hundred and Fifty $\frac{00}{100}$ Dollars for value received, with interest at 5% per annum.

J. D. Crawford.

The person who signs the note (J. D. Crawford) is called **the maker.**

The person to whom the money is to be paid (James H. Broadbent) is called the **payee.**

The sum borrowed ($250) is called the **face of the note.**

When dates are given, interest for a part of a year is usually calculated by days. For instance, a $250 account due on 1st November but not paid until 1st February would be 92 days overdue, and interest on it for that period at 5% would, therefore, be $\frac{92}{365}$ of the interest for one year, or $\frac{92}{365}$ of $\frac{5}{100}$ of $250.

EXERCISE 82

1. Find the interest on $640 for 3 years at $5\frac{1}{2}\%$ per annum. (Interest $= 3 \times \frac{11}{200}$ of $640.)

2. Find the interest on $200 for 1 year at 6% per annum; $350 for 2 years at 4% per annum; $50 for $2\frac{1}{2}$ years at 3% per annum; $280 for $1\frac{1}{4}$ years at $3\frac{1}{2}\%$ per annum; $75·50 for $6\frac{2}{3}$ years at $4\frac{1}{2}\%$ per annum.

3. Find, to the nearest cent, the interest on, and the amount of, $428·25 for 2 years and 3 months at 6%. (Interest $= \frac{27}{200}$ of principal, therefore amount $= \frac{227}{200}$ of principal.)

4. Find, to the nearest cent, the interest on, and the amount of, $562·85 for 3 years and 73 days at 5%; $4200 from 5th May, 1909, to 8th September, 1909, at $4\frac{1}{2}\%$; $3725 from 10th June, 1909, to 8th May, 1910, at $3\frac{1}{2}\%$; $6250 from 8th January, 1910, to 6th March, 1911, at 4%; $10000 from 30th April to 8th October at $5\frac{1}{2}\%$.

5. On what principal is the interest $45 in $2\frac{1}{2}$ years at 6%? ($\frac{15}{100}$ of principal = interest.)

6. On what principal is the interest $60 in 2 years at 6%? $144 in $1\frac{1}{3}$ years at $4\frac{1}{2}\%$? $25·50 in 6 months at $4\frac{1}{4}\%$? $198 in 90 days at $5\frac{1}{2}\%$?

7. In what time will $250 give $75 interest at 6%? (Interest for 1 year $= \frac{6}{100}$ of $250 = $15.)

8. In what time will \$625 give \$12·50 at 4% ? \$4280 give \$748 interest at 5% ?

9. Find the rate per cent. per annum when the interest on \$840 for $2\frac{1}{2}$ years is \$105. (Interest for 1 year = \$42 = $\frac{42}{840}$ of principal = 5% of principal.)

10. Find the rate per cent. per annum when the interest on \$6000 for 1 year is \$240; on \$1600 for 9 months is \$72; on \$2190 for 80 days is \$16·80.

11. At what rate per cent. per annum will \$1000 amount to \$1030 in 8 months ? \$620 to \$713 in 3 years ? \$360 to \$403·20 in 4 years ?

12. On 5th January, 1910, Mr. Jackson borrowed \$250 and agreed to pay interest thereon at $4\frac{1}{2}$% per annum. How much did he owe on 5th April, 1910 ?

13. On 6th June, 1910, Thomas Black, of Sarnia, got from Henry Stone goods valued at \$125 and agreed to pay for them in four months with interest at 6% per annum. On what day was the account due ? How much did Mr. Black owe Mr. Stone on that day ?

14. Write a receipt for the interest on a loan of \$80 for 5 months with interest at $5\frac{1}{2}$% per annum.

15. If you deposit in a savings-bank \$120 on 1st January, and \$180 on 3rd February, how much should there be to your credit on 31st March if the bank allows you interest at the rate of $3\frac{1}{2}$% per annum ?

16. A savings-bank which pays interest at the rate of 4% per annum makes up its interest on 30th June. If the interest on any account is equal to the difference between the interest up to 30th June on all sums withdrawn and the interest on all sums deposited, find the amount of Henry Rust's account on 30th June if on 10th January he deposits \$200, on 15th February withdraws \$150, on 25th February deposits \$250, on 4th April deposits \$400, on 8th May withdraws \$200, on 30th June deposits his interest.

17. On 6th January a customer buys goods valued at $136, but does not pay his bill until 4th June. How much should he then pay if the merchant charges 4% interest on all accounts ?

18. On 4th September I buy goods worth $200, and am allowed 30 days in which to make payment. After that time I agree to pay 3% interest. How much shall I owe on 10th December ?

NOTE: On demand notes, payments may be made at any time. The amount of the note at the date of each payment is found, the payment is deducted from that amount, and the balance bears interest until the next payment, and so on.

19. On 1st February, 1910, Mr. Brown gave the Bank of Commerce a demand note for $2000 bearing interest at 6%. Counting by months, what would be the amount due on 1st April, 1910 ? If Mr. Brown then paid $400, what would the balance be ? What would be the amount of this balance on 1st June, 1910 ? If Mr. Brown then paid $300 what would the balance be ? What would be the amount of this balance on 1st September, 1910 ? If Mr. Brown then paid $900, what would the balance be ? What would be the amount still due on 1st December, 1910 ?

TRADE DISCOUNT

It is a custom among certain manufacturers and wholesale dealers to issue catalogues containing a list of prices as well as a description of the goods they are offering for sale. These catalogues are intended for the use of the *trade*, that is, for the retail merchants who handle the merchandise described. The prices given are called the **list** or **catalogue prices,** and a reduction from them is usually allowed to the trade. This reduction is called *trade* or *commercial discount*, and is quoted as a percentage of the *list price.*

When the manufacturer or wholesale merchant wishes to change his prices he does so either by changing the *rate* of *discount*, or by giving *additional discounts.*

If two or more discounts are allowed, the first one is a discount off the list price, the second off the remainder, the third off what then remains, and so on.

The price remaining after all discounts are deducted is called the **net price.**

Retail merchants fix for their goods a *selling price,* just as the wholesale merchant has a *list price* for his goods. This is usually called the **marked price,** and is a percentage of the **cost price.** Customers are sometimes given a discount off this *marked price.*

EXERCISE 83

1. Find the discount from each of the following list prices at the rate given: \$1 with 10% off, \$4 with 25% off, \$10 with 30% off, \$9 with $33\frac{1}{3}$% off, \$15 with 5% off, \$25 with 20% off, \$40 with $12\frac{1}{2}$% off, \$75 with 8% off, \$120 with $37\frac{1}{2}$% off, \$600 with 40% off. *Solve all these mentally.*

2. Find the *net* price for each *list* price in the preceding example.

3. Find the discount and net price for each of the following amounts at the rate given: \$12·50 with 10% off, \$15·25 with 8% off, \$25·75 with 20% off, \$50 with 15% off, \$225 with 30% off, \$626 with 25% off, \$4250 with $37\frac{1}{2}$% off, \$637·25 with 4% off, \$982·80 with $33\frac{1}{3}$% off, \$3264·80 with 3% off.

4. Find the net price of goods listed at \$400 with discounts of 30% and 10% off. (Net price $= \frac{9}{10}$ of $\frac{7}{10}$ of \$400.)

5. The net price of an article listed at \$16 was \$12·80. What was the rate of discount? (Discount = \$3·20 $= \frac{320}{1600}$ of the list price = 20%.)

6. Find the net prices for each of the following: \$625 with 20% and 10% off, \$3275 with 40% and 5% off, \$4000

with 25% and 20% off, \$1640 with $37\frac{1}{2}$% and 10% off, \$2160 with 15% and 5% off.

7. Find the net prices for: \$800 with 30%, 10% and 5% off; \$680 with 20%, 20% and 3% off; \$4650 with $33\frac{1}{3}$%, 10% and 5% off; \$3400 with 10%, 10% and 10% off; \$765 with 20%, 10% and 5% off.

8. A customer paid \$3·60 for an article of which the marked price was \$4. What rate of discount did he receive?

9. Goods for which the list price was \$90 were sold to a retail merchant for \$58·50. What was the rate of discount?

10. The list price is \$500. What is the difference between a discount of 40% and two discounts of 20% and 20%?

11. The catalogue price of certain goods was \$1575, off which there were given two discounts of $33\frac{1}{3}$% and 6%. How much less would they have cost if there had been a single discount of 40%?

12. A merchant buys from a manufacturer cloth which is catalogued at \$3·20 a yard. He gets a discount of 25% and sells the cloth at a gain of 20%. At what price did he sell it?

13. What is the list price of knives for which a retail merchant pays \$3 a dozen after getting a discount of $33\frac{1}{3}$%?

14. A wholesale merchant gives 25% and 10% discount. What was the list price of goods for which he received \$445·50?

15. A retail bookseller to whom the wholesale merchant allows discounts of 20% and 15%, buys a book of which the catalogue price is \$2.50 and sells it at a profit of 30%. What does he get for it?

16. A merchant, who sells goods on 60 days' credit, allows 3% discount off all bills for cash. How much does a customer save by paying cash for a bill of \$265?

17. A merchant allows a customer 5% off his bill for cash. What is the amount of a bill for which a customer pays \$39·90 cash?

18. If, in the preceding question, the merchant was marking his goods at a profit of $33\frac{1}{3}\%$, what was the cost of the goods for which he received $39·90?

19. A dealer sold a plate glass for $150, less discounts of 20%, 10%, and 5%, and made a profit of 20%. What did the plate glass cost him?

20. Toronto, 17th Dec., 1909.

Mr. J. D. Winch,
Orillia.

Bought of The Furniture Co.

Net—60 days.
Terms—3% discount if paid in 30 days.

		$	c.	$	c.
24	Kitchen Chairs (№ 795) @ $ 2·50	60	00		
1	Kitchen Cupboard (№ 524) @ $20·50	20	50		
2	Extension Tables (№ 150) @ $18·00	36	00		
1	Hall Rack (№ 369) @ $17·50	35	00		
		151	50		
	Less 40%, 10%	69	69		
				81	81

NOTE: № means *number*. Thus № 795 means No. 795 in the catalogue issued by The Furniture Co.

Is the above bill correct? What amount would settle the bill on 29th December, 1909?

Supply names and dates, and make out bills in the above form for the following:

21. Bought 3 account books @ $3·50 each, 5 doz. exercise books @ 25c. each, 10 Morocco bound notebooks @ $1·25 each, 5 gross penholders @ 60c. per doz., 4 double inkstands @ $1·25 each. Discounts 20% and 5%.

22. Bought 4 diamond rings @ $35 each, 6 gold lockets @ $6·50 each, 3 watches @ $30 each, 1 silver-plated tea-set @ $35, 2 cut glass vases @ $8·50 each. Discounts 30% and 10%.

23. Bought 30 half-pound tins spice @ 7c., 25 lb. baking powder at 8c., 20 lb. ground pepper @ 35c., 150 lb. coffee @ 40c., 200 lb. Japan tea @ 35c., 100 lb. rice at 6c., 12 boxes raisins @ $4. Discounts 25% and $12\frac{1}{2}\%$.

COMPOUND INTEREST

If Mr. Longford borrows from Mr. Sinclair \$200 with interest at 5% per annum, he will owe at the end of the year \$200 and \$10 interest. If now he is unable to pay the interest, he is really borrowing \$210 from Mr. Sinclair for the second year, the interest on which will be \$10·50, and the amount which Mr. Longford will owe at the end of two years is \$220·50. The difference between this amount and the original principal is called the **compound interest** on that principal for two years. That is, \$20·50 is the compound interest on \$200 for 2 years at 5% per annum.

The ordinary interest on \$200 for 2 years at 5% is \$20, which is called the **simple interest**. In what respect does compound interest differ from simple?

The method of finding compound interest is the same as that used for simple interest.

Example: Find the amount of, and the interest on, \$1500 for $1\frac{1}{2}$ years at 4% per annum, the interest to be compounded semi-annually:

FIRST SOLUTION

\$1500 = first principal.
·02
\$30 = interest for first $\frac{1}{2}$ year.
1500
\$1530 = amount after first $\frac{1}{2}$ year.
·02
\$30·60 = interest for second $\frac{1}{2}$ year
1530·00
\$1560·60 = amount after 1 year.
·02
\$31·21 = interest for third $\frac{1}{2}$ year.
1560·60
\$1591·81 = amount after $1\frac{1}{2}$ years. Interest = \$91·81.

SECOND SOLUTION

Amount after any ½ yr. = 1·02 × principal at the beginning of that ½ year.

Amount at the end of first ½ yr. = 1·02 × first principal
= second principal.

Amount at the end of second ½ yr. = 1·02 × second principal
= 1·02 × 1·02 × first principal
= third principal.

Amount at the end of third ½ yr. = 1·02 × third principal
= 1·02 × 1·02 × 1·02 × first principal
= 1·061208 × $1500
= $1591·81

Interest for 1½ yr. = $1591·81 − $1500 = $91·81.

NOTE: For brevity it is usual to write $1·02^2$ instead of 1·02 × 1·02 and $1·02^3$ instead of 1·02 × 1·02 × 1·02, and so on. The small figure placed above and to the right of the number indicates the number of times the factor is repeated, thus: $3 \times 3 \times 3 \times 3 \times 3$ is written 3^5.

Since the amount at the end of 1½ yr. = 1·061208 × principal, what fraction of the principal must the interest be?

The amount at the end of each half-year may be written as follows:

The amount at the end of first ½ yr. = 1·02 × principal.
The amount at the end of second ½ yr. = $1·02^2$ × principal.
The amount at the end of third ½ yr. = $1·02^3$ × principal.

What would represent the amount at the end of 6 years (12th half-year)? At the end of 7 years? At the end of 10 years? At the end of 5½ years? At the end of 8½ years?

What was done in the second solution above was to find the amount of, or the interest on, $1 for the given time at the given rate, and multiply this by the number of dollars in the principal.

EXERCISE 84

(In these exercises find results to the nearest cent.)

1. Find the amount and the compound interest of \$500 for 3 years at 4%. [Interest $= (1{\cdot}04^3 - 1) \times \$500$.]

2. Find the amount and the compound interest of \$200 for 2 years at 5%; \$3000 for 4 years at 6%; \$2000 for 3 years at $5\frac{1}{4}$%; \$4800 for 4 years at $3\frac{1}{2}$%; \$6000 for 4 years at $4\frac{1}{4}$%.

3. Find the amount and the compound interest of \$300 for 2 years at 4% compounded half-yearly. (Amount $= 1{\cdot}02^4 \times \$300$.)

4. Find the amount and the compound interest of \$800 for $1\frac{1}{2}$ years at $3\frac{1}{2}$% compounded half-yearly; \$5000 for $2\frac{1}{2}$ years at 5% compounded half-yearly; \$2000 for 1 year at 4% compounded quarterly.

5. Find the amount of \$1 in 4 years at 5% compounded yearly. Use the result to find: (*a*) The amount of \$650 for 4 years at 5% compounded yearly; (*b*) the sum which in 4 years at 5% compounded yearly will amount to \$7250.

6. Find the principal which will amount to \$3800 in $1\frac{1}{2}$ years at 5% compounded half-yearly; \$5200 in 2 years at 6% compounded half-yearly; \$4750 in 1 year at 4% compounded quarterly.

7. A savings-bank which pays 4% per annum compounds its interest half-yearly on 30th June and 31st December. How much will a man have to his credit on 31st December who deposits \$300 on 1st February, and \$900 on 1st August?

8. Find the difference between the simple interest and the interest compounded half-yearly on \$1 for $1\frac{1}{2}$ years at 6%. Use the result to find:

(*a*) The difference between the simple interest and the interest compounded half-yearly on \$900 for $1\frac{1}{2}$ years at 6%.

(*b*) The sum for which the difference between the simple interest and the interest compounded half-yearly is \$28·30 for $1\frac{1}{2}$ years at 6%.

9. What is the difference between the simple interest and the interest compounded yearly on \$3600 for 2 years at $3\frac{1}{2}\%$.

INSURANCE

Every year much property is destroyed or damaged by fire. So seriously do most persons fear the loss which they might suffer from this cause that it is now an almost universal custom for property owners to enter into an agreement with certain companies by which, in return for a stated annual payment, the companies will pay a specified sum for any loss sustained.

These companies are called *Fire Insurance Companies.* The agreement to compensate for loss is called **insurance.**

The written contract between the property owner and the company is called a **policy.**

The sum specified to be paid in case of loss is called the **amount** or **face** of the policy, or the **risk.**

The cost of insurance, that is, the payment made by the property owner to the company, is called the **premium,** and it is a percentage of the risk.

Rates for insurance vary for different locations, for different businesses, for different kinds of property. Why?

EXERCISE 85

1. What is the premium on a \$500 policy at $\frac{3}{4}\%$?

2. What is the premium on a \$2200 policy at 65c. for \$100 ?

3. An hotel worth \$12500 is insured for $\frac{4}{5}$ of its value at $1\frac{1}{2}\%$. What is the premium ?

4. Find the face of the policy on which \$19·20 is paid as a premium at $\frac{3}{4}\%$.

5. What amount of risk can I get for \$25·60 at 80c. per \$100 ?

6. A dealer insured his stock of goods for \$64000, paying as premium \$96. Find the rate of insurance ?

NOTE: What fraction is the premium of the amount of the policy? What per cent.?

7. A manufacturer insured his factory valued at \$12000 for $87\frac{1}{2}\%$ of its value and paid a premium of \$280. What was the rate of insurance ?

8. A building worth \$6000 is insured for \$4500 at 9%. In case the building is destroyed, what will be the owner's net loss ?

9. A factory worth \$8000 is insured for \$5000 at $2\frac{1}{2}\%$, and is damaged by fire to the extent of \$6000. What is the owner's net loss ?

10. A farmer insures his house for \$1200 and its contents for \$750, his barn for \$1400 and its contents for \$800. Find the total premium paid at $\frac{4}{5}\%$.

11. If brick houses are insured for 75c. a \$100 while frame houses are charged 1%, what is the difference between the premium paid for insuring a brick house and a frame house for \$1840 each ?

12. Mr. Leslie insured his house for $\frac{9}{10}\%$ and found that in case of loss he would recover the value of the building as well as the premium paid. If the premium was \$45, what was the value of the house ?

13. For what sum must grain valued at \$2485 be insured at $\frac{8}{5}\%$ so that in case of loss the owner may recover the value of the grain together with the premium paid ?

14. Mr. Hay insures at $1\frac{1}{4}\%$ his goods invoiced at \$23800 for 80% of their value. If they should be destroyed by fire, what would be Mr. Hay's total loss ?

15. A stock of goods is insured in the London Assurance Co. for \$5500, in the Queen Assurance Co. for \$3500, and in the Mutual Assurance Co. for \$6500. If the goods are damaged to the extent of \$9000, how much of the loss will each company have to pay ?

16. An insurance company which charged a rate of $\frac{4}{5}\%$ received \$60 for insuring a house for $\frac{3}{4}$ of its value. Find the value of the house.

17. Is your house insured? in what company? for what sum? at what rate? Read the policy and calculate the premium to be paid.

BANK DISCOUNT

A great deal of a bank's business consists in discounting notes, that is, in cashing or paying them before they are due.

If a person, say, Mr. Sidney Johnston, wishes to borrow money from a bank, he may do so on one or more of several kinds of notes, but the form of the one commonly used is as follows:

$1000.00 Chatham, 10 Dec., 1909.

Ninety days after date I promise to pay to the order of Timothy Newton One Thousand $\frac{00}{100}$ dollars at the Standard Bank, Chatham.

Value received. Sidney Johnston.

Mr. Newton endorses this note by writing his name across the back of it, and so becomes responsible for its payment by Mr. Johnston or by himself.

Mr. Johnston, or Mr. Newton for him, may then take the note to the Standard Bank and receive cash for it, provided the bank officials are satisfied that both men are reliable.

The bank, however, will not pay the full amount of $1000 but will *deduct* a percentage of it; and the sum so deducted is called **bank discount.**

If the note is presented at the bank on 10th December, 1909, and the bank's rate of discount is 7% per annum, the discount will be $\frac{93}{365}$ of $\frac{7}{100}$ of $1000, or $17·84.

The amount which the bank would pay on the note would be $1000 − $17·84, or $982·16, which is called the **proceeds** of the note.

In what way does bank discount resemble trade discount?

Although the time named in the note, that is, the *nominal* time, for payment is 90 days, the discount for 93 days is taken, because in Canada a note is not **legally** due until *three days* after it is **nominally** due.

These three extra days are called **days of grace,** and banks always include them in the term of discount.

If the note is not presented at the bank until, say 1st January, 1910, the bank will then deduct only 73, that is 93 − 20, days' discount.

The number of days from the day on which the bank gets a note to the day on which the note is legally due is called the *unexpired time* or the *term of discount.*

Since the above note was not worth $1000 until it was due, this sum is called the *value of the note at maturity.* In interest-bearing notes the value at maturity includes the interest due on the **legal** date for payment.

The bank discount on a note is *a percentage of its value at maturity for the term of discount at the bank rate of discount.*

The proceeds are the value at maturity less the bank discount.

Look again at the note on page 173. What was the $250 called? What was the value of this note at maturity? On what day was it *legally* due? If Mr. Broadbent sold it to the bank on 6th January, 1910, what would be the term of discount? If the bank's rate of discount is 6%, what would be the discount? What the proceeds?

EXERCISE 86

1. On 5th January, 1910, William Lynch gives to the Bank of Montreal his note, endorsed by John Lawrence, for $300 payable in 3 months without interest. Write the

note, dating it 5th January, 1910. On what day was it legally due? Find the proceeds if the bank's rate of discount is 6%. (Bank discount $= \frac{93}{365}$ of $\frac{6}{100}$ of \$300. Proceeds = \$300 − bank discount.)

Find the day of maturity, the term of discount, the discount, and the proceeds of the following:

2. A 60-day note dated 4th October, 1909, for \$400 without interest and discounted on the same day at 5% per annum.

3. A 90-day note dated 3rd May, 1910, for \$1000 without interest and discounted immediately at $5\frac{1}{2}$%.

4. A 4-month note dated 8th June, 1910, for \$6000 without interest and discounted 4th July, 1910, at 5%.

5. A 90-day note dated 6th July, 1909, for \$6000 without interest and discounted 26th July, 1909, at $5\frac{1}{2}$%.

6. A 6-month note dated 10th June, 1910, for \$1500 without interest and discounted 20th July, 1910, at 6%.

7. If a 60-day note without interest is immediately discounted at 7%, what fraction of its value at maturity would the discount be? What fraction of its value at maturity would the proceeds be? If the proceeds were \$3605·90, what would be the value of the note at maturity?

8. For what sum must a 90-day note be drawn so that when it is discounted immediately at 5% the proceeds may be \$1441·40?

9. For what sum must a 3-month note dated 10th May, 1910, be drawn so that if it is discounted on 2nd June, 1910, at $6\frac{1}{4}$% the proceeds may be \$3605?

10. On 6th February, 1908, R. T. Sutton borrowed \$600 from M. L. McBain on a 6-month note bearing interest at 5% per annum. Write the note. On what day was it legally due? What was its value on the day of maturity?

11. If Mr. McBain endorsed the note in the preceding problem and sold it to the Royal Bank on 26th May, 1908, what did he get for it if the bank's rate of discount was $5\frac{1}{2}$%?

Find the day of maturity, the value at maturity, the term of discount, the discount, and the proceeds of the following:

12. A 90-day note dated 17th May, 1910, for $2500 with interest at 5% and discounted at the bank on 3rd June, 1910, at 7%.

13. A 4-month note dated 14th August, 1908, for $1200 with interest at 5% and discounted at the bank the same day at 5%.

14. A 60-day note for $3650 without interest realized $3608 when discounted at the bank on the day it was drawn. What was the value of the note at maturity? What was the term of discount? What was the discount for this term? What would be the discount for one year? What fraction is the year's discount of the value at maturity? What, therefore, is the rate of discount?

15. A 3-month note for $365 dated 1st July, 1909, and bearing interest at the rate of 4% yields $364·19, when discounted at the bank on 23rd July, 1909. Find the bank's rate of discount.

STOCKS

A man who goes into business for himself invests in it an amount of money which is called his **capital.** At regular periods he compares his total receipts with his total expenditures, and the excess of the former over the latter is called his **profits** on the invested capital.

If the man has a partner, the two men form what is commonly called a *firm*, and the required capital is obtained by each man furnishing a part of it. *The profits are divided between them according to the amount of money each invested.* The same course would be followed if there were several partners.

It often happens, however, that a business undertaking may be too large for one or two men to supply all the

money which may be needed, and so a number of persons unite and form what is called a *stock company* or *corporation.* They arrange to raise the money necessary for the business by fixing the *capital* of the company at a settled amount, not of *money* but of *stock.* This stock is then divided up into an equal number of shares, and each share is *valued* at a fixed amount, usually $100, which is called the **par value** of a share of stock. These shares are then sold.

A person becomes a member of the company by buying one or more of its shares of stock. He is then a *stockholder* or *shareholder* and is given a *stock certificate.*

The **profits** of the company are called its **dividend** and are divided at regular periods among the shareholders *according to the number of shares* each possesses.

Stock is not money, but it can be bought and sold for money, and a shareholder can get money for his stock only by selling it to some person who is willing to buy. The price paid for it will depend largely upon the success of the company and upon the amount of the dividend.

Although as stated above, stock is usually divided into $100 shares, there are stocks with shares of different amounts, such as $1, $5, $10, $50, $200.

With the exception perhaps of some mining stocks, the market value of the shares is quoted in stock-lists, newspapers, and elsewhere as a percentage of the par value. Thus, Bank of Toronto stock listed at 215 means that a $100 share sells for $215.

Stock is at *par,* at a *discount,* or at a *premium,* according as its shares sell *for, below,* or *above* their par values. The market value of stock must be clearly distinguished from its par value.

Stock is generally bought and sold through a *stock-broker.* He charges for his services a percentage of the par value of the stock. This charge is called **brokerage.**

EXERCISE 87

1. What is the par value and the market value of 25 shares of Dominion Bank stock at 241 ? of 60 shares of Dominion Steel stock at $74\frac{1}{4}$? of 57 shares Toronto Railway at $126\frac{1}{2}$? of 23 shares Lake Superior at 27 ?

2. How many shares can be bought of Twin City Railway stock at 113 for \$3,390 ? of Imperial Bank stock at 230 for \$9,430 ? of Mexican Light and Power stock at $66\frac{1}{2}$ for \$3,657·50 ? of Bell Telephone stock at $146\frac{1}{2}$ for \$4,248·50 ?

3. What is the annual dividend obtained from 36 shares of stock which pays 6% ? from 92 shares of stock which pays $12\frac{1}{2}$% ? from 44 shares of stock which pays $3\frac{1}{2}$% half yearly ? from 85 shares of stock which pays 5% half yearly ?

4. How many shares of the par value of \$100 will there be in a total capital of \$50000 stock ? of \$150000 stock ? of \$250000 stock ? of \$3000000 stock ?

5. What will be the dividend on each share of stock when the capital and total profits are respectively, \$20000 and \$1000 ? \$250000 and \$11250 ? \$750000 and \$52500 ? \$1500000 and \$123750 ?

6. What is the rate of the dividend when :

(*a*) \$11000 stock gives an annual income of \$495 ?
(*b*) \$6200 stock gives an annual income of \$434 ?
(*c*) \$9000 stock gives an annual income of \$382·50 ?
(*d*) \$7500 stock gives an annual income of \$400 ?

7. The holder of 58 shares of railway stock sells at 65 and invests the proceeds in a bank-stock at 145. How many shares of bank-stock does he buy ?

8. A man invests \$6150 in a bank-stock at 123. If the stock pays a dividend of 7% per annum, what is the man's income from his stock ?

9. Mr. Thornton instructs a broker to buy for him 25 shares of Lake of the Woods stock at $135\frac{3}{4}$. If the broker charges $\frac{1}{4}\%$ commission, what does the stock cost Mr. Thornton? How much does the broker receive?

10. I instruct my broker to sell for me 18 shares of Nova Scotia Coal stock at 69. If he charges $\frac{1}{2}\%$ brokerage, what do I receive for the stock?

11. A man invests \$3,780 in stock at $94\frac{1}{4}$ and sells when the stock rises to $110\frac{3}{4}$? How much does he gain by the transaction, if he has to pay $\frac{1}{4}\%$ brokerage for buying and the same for selling?

12. What annual income will be obtained from investing \$10,010 in a 6% stock at 125, brokerage being $\frac{1}{8}\%$?

13. The holder of \$7,200 stock which pays an annual dividend of 5% sells at $96\frac{1}{2}$ and invests the proceeds in an 8% stock at $119\frac{1}{2}$. What will be the difference in his annual income, brokerage for buying and for selling being $\frac{1}{2}\%$ in each case?

14. A man invests \$3,820 in a 3% stock at $95\frac{1}{2}$, \$6,000 in a 4% stock at par, and \$8,660 in a 5% stock at $108\frac{1}{4}$. How many shares of stock has he? What is their total par value? What is his total income from his stocks?

15. Mr. Riddell invests \$4,800 in a 6% stock at 120. How many shares of stock does he buy? What will be his annual income? What fraction is this income of the money invested? What per cent. therefore, is the rate of interest on the money invested?

16. If I invest \$5000 in an 8% stock at 125, what rate of interest do I receive on the money invested?

17. If I pay \$75 for a share of 6% stock, what rate of interest shall I get on the money invested?

18. A man has \$4200 which he can loan at $5\frac{1}{2}\%$ per annum or invest in a 6% stock at 105. How much larger will his income be by taking the better investment?

19. How much money must be invested in a $7\frac{1}{2}\%$ stock at 115 to give an annual income of \$300?

20. How much 4% stock must I have to give me an income of $360? What would it cost at 80?

21. A man who invests in a 10% stock finds that he is getting $6\frac{1}{4}$% interest on the money invested. What is the price of the stock?

22. A man who wishes to make 5% interest on his money invests it in a $7\frac{1}{2}$% stock. What price per share would he be willing to pay for the stock?

23. A man invested $6300 in a $3\frac{1}{2}$% stock at $104\frac{3}{4}$ and sold at $108\frac{3}{4}$. He then invested the proceeds in a 6% stock at $162\frac{1}{2}$. Find the change in his income, if the brokerage for buying and selling was $\frac{1}{4}$% in each case.

24. How much would the broker receive for the three transactions in the preceding problem?

25. By investing in a 6% stock, Mr. Brown finds that he is getting 8% interest on the money invested. What did he pay for the stock?

26. Look up the stock-market reports in your daily papers, and find the amount of a day's sales for any one stock. Find, also, the amount of the brokerage at $\frac{1}{4}$% paid on a day's transactions in any one stock.

27. Two men form a partnership to conduct a dry goods store. One invests $5000 and the other $7000. If their yearly receipts amounted to $15000 and their expenses to $14100, what rate per cent. dividend can they pay on the money invested? What will be each man's share of this dividend?

28. Three men formed a partnership in which they invested $6000, $7500, and $9000 respectively. Their annual profits were $2500 of which all but 10% was paid as a dividend to the partners. What was the rate of the dividend, and what was each man's share?

29. A, B and C formed a partnership, their total investments being $20000. After paying expenses which were 20% of their total gains, A received $1,280 net, B $1,120, and C $800. Find the amount invested by each and the amount of expenses borne by each.

EXCHANGE

The buying and selling of goods is constantly going on between all parts of the world, and as a result it is necessary to have some means by which the payment of money can be made between places distant from each other.

The settling of accounts between persons residing at a distance from each other by means of written orders is called **exchange.** These written orders, such as cheques, drafts, express money-orders, etc., are called **bills of exchange.**

If Mr. James Kay, living in Peterborough, owes $50 to Mr. William Wye, living in Ottawa, in what different ways may Mr. Kay pay his debts?

Paying by cheque : In the example above, if Mr. Kay has money to his credit in any bank, he may make out a cheque for $50. Write the cheque.

This cheque he will send to Mr. Wye who can get it cashed at some bank in Ottawa. The bank will likely deduct a small sum from the face of the cheque.

Paying by bank draft : Instead of sending the above cheque Mr. Kay might go to a bank and buy a **bank draft.**

This draft is sent by Mr. Kay to Mr. Wye who gets it cashed for its *full* value in Ottawa. Mr. Kay would likely have to pay for the draft a little more than $50.

Commercial draft : The account between Mr. Kay and Mr. Wye may also be settled by Mr. Wye making a draft on Mr. Kay. Write the draft.

Some bank in Ottawa pays Mr. Wye on this draft $50, less the cost of exchange, and sends the draft to a bank in Peterborough which collects the money from Mr. Kay.

The sum to be paid for drafts or bills of exchange is usually a certain per cent. of their face value, and the difference in percentage between the face value and the amount paid is called the **rate of exchange.**

Post-Office, Express, and **Bank Money-orders** are in form much like bank drafts. They are bought and cashed at post-offices, Express Companies' offices, and at banks, respectively. They are sold for a slight advance over their face values.

For Post-Office Orders issued in Canada, for payment in Canada and United States, the advance above the face value is as follows:

For $5 and under, 3c. Over $30 and up to $50, 15c.
Over $5 and up to $10, 6c. Over $50 and up to $75, 25c.
Over $10 and up to $30, 10c. Over $75 and up to $100, 30c.

For small sums the post-office also sells **postal notes** of the following denominations:

(1) 20c., 25c., 30c.. 40c., and 50c., each of which costs 1 cent extra;

(2) 60c., 70c., 80c., 90c., $1, $1·50, $2, and $2·50, each of which costs 2 cents extra; and

(3) $3, $4, $5, and $10, each of which costs 3 cents extra.

The odd cents are made up by attaching postage-stamps to the note; for instance, 38 cents would be made up by a 30c. *postal note* and 8c. in postage-stamps.

The extra charges for express and bank money-orders up to $50 are the same as those for Post-Office Orders.

In case of Foreign Exchange, that is, exchange between two places which are in different countries, we shall have to consider not only the rates of exchange, but also the value of the coins used in the two places.

The values of the coins of a few countries are here given:

Country	Coin	Canadian Value
Great Britain..	1 pound	$4·86⅔
France..... Belgium..... Switzerland..	1 franc (100 centimes)	19·3 cents
Germany.....	1 mark (100 pfennige)	23·85 cents

EXERCISE 88

1. At the rates quoted above, find the cost of post-office money-orders for: $4·10, $14, $26·50, $30·08, $47·75, $74·99, $93·30.

2. For each of the following sums what postal notes would you buy, and what would be the total cost in each case: 45c., 95c., $1·65, $2·93, $3·79, $4·85, $5·92, $9·15, $26?

3. What is the difference in cost between money-orders and postal notes for: 85c., $3·30, $6·40, $18, $23·75, $28, $30, $40·15?

4. The rate of exchange being ⅛%, find the cost of a bank draft for $360, $2000, $236·50.

5. Mr. James Wilkinson, St. Thomas, buys $1200 worth of goods from Brown Bros., Toronto, and sends them a draft, issued by the Merchants Bank, for the amount. Write the bank draft. What will it cost Mr. Wilkinson, exchange being at ⅛%?

6. The Atlantic Lumber Co., Halifax, sold $2400 worth of lumber to R. Higgins & Co., Montreal, and drew on them for the amount through the Bank of Nova Scotia. Write the commercial draft. How much would the Lumber Co. get from the bank, exchange being at ¼%?

7. Mr. F. Davis, of Simcoe, sends Mr. R. Carter, Belleville, a cheque on the Bank of Hamilton in payment of an account of $160. Write the cheque. What will Mr. Carter get for it, exchange being at ⅛%?

8. What will a Toronto merchant pay for a draft on Chicago for $3200, exchange being at $\frac{1}{4}\%$?

9. What will be the face of a draft for which $720·90 is paid when exchange is at $\frac{1}{8}\%$?

10. What must be the face of a cheque so that when it is cashed at a bank the proceeds will be sufficient to pay a bill for $319·20, the bank's charge for exchange being $\frac{1}{4}\%$?

11. What must be the face of a cheque so that when the bank's charges of $\frac{1}{2}\%$ for exchange are deducted, the proceeds will be sufficient to pay a bill for $139·30 ?

12. When $1565·20 is paid for a $1560 draft, what is the rate of exchange ?

13. What is the value in Canadian money of: £1240, 3500 francs, 4800 marks ?

14. $6400 is equal to how many pounds ? $4530 to how many francs ? $8000 to how many marks ?

EXERCISE 89

1. A man buys two farms. For the one, containing 65 acres, he paid $48 an acre, and for the other, containing 85 acres, he paid $64 an acre. If he sells the first at a gain of $12\frac{1}{2}\%$ and the second at a loss of $6\frac{1}{4}\%$, will he gain or lose on the whole, and how much ?

2. A banker receives on a certain day payments for the following: $360 and interest for 3 mo. at 6% ; $150 and interest for 1 yr. at 5% ; $800 and interest for 4 mo. at $4\frac{1}{2}\%$; $250 and interest (simple) for $1\frac{1}{2}$ yr. at 4%. Write a receipt for the full amount paid in each case.

3. In a certain season a ball team won 54 games and lost 16. Find its winning average, that is, the per cent. of games won.

4. A gentleman bought a $1200 automobile at 15% discount, and after using it for two seasons sold it for $33\frac{1}{3}\%$ less than he paid for it. How much did he receive for it ?

5. Three persons are partners in a business in which the first invests \$1200, the second \$1600, and the third \$800. What per cent. of the business does each own? How should a profit of \$2400 be divided among them?

6. Mr. Saunders insures a stock of goods worth \$14,000 for 80% of its value at $1\frac{1}{8}$%. If the goods are entirely destroyed by fire, what is the loss, including the premium paid?

7. A draft cost a merchant \$1,515. The rate of exchange being 1%, what was the face of the draft?

8. A manufacturer sells an article for \$3·24. The material cost him \$1·55, the labour 55c., and other expenses 40c. Find his gain per cent.

9. For selling some property on a commission of $2\frac{1}{2}$% an agent received \$29·50. What did he receive for the property? What did his employer receive for it?

10. A dealer bought 6000 tons of coal at \$4 a ton. He sold $33\frac{1}{3}$% of it at an advance of 25%. He then sold 20% of the remainder at an advance of 20%, and the rest he sold at \$4·75 a ton. Find his total gain.

11. What would be the compound interest in two years on \$1400 at 4% half-yearly. What would be the simple interest on the same sum for the same time at $4\frac{1}{2}$% per annum?

12. An agent receives \$1,624 to invest after deducting a commission of $1\frac{1}{2}$% on the amount invested. Find his commission.

13. The sum of the principal and the interest for 4 mo. at 6% per annum was \$326·91. Find the principal.

14. At $\frac{1}{2}$c. a lb. and 35% ad valorem, what will be the duty on 280 lb. of candied orange-peel invoiced at 15c. a lb.?

15. Make out a 90-day note for \$600, dated to-day and payable to Henry Martin at your nearest bank. Discount it immediately at 6% and find the proceeds.

16. A roll of 120 yd. of cloth was sold for \$216 at a loss of 10%. For what should it have been sold a yard to give a profit of 20%?

17. On a demand note for $1000, dated 15th Jan., 1910, and drawing interest at 6%, the following payments were made: 26th Feb., 1910, $125; 28th March, 1910, $220; 17th June, 1910, $400. How much was due 6th Sept., 1910?

18. A premium of $66 is paid for insuring a cargo of grain for $\frac{3}{4}$ its value. If the rate of insurance is $1\frac{1}{4}\%$ and the grain worth 80c. a bushel, find the number of bushels in the cargo.

19. How much tax must be paid by a man whose property is assessed for $8500, if the rate is $6\frac{1}{2}$ mills?

20. Mr. Edwards buys 100 shares of stock at 106, and after receiving 4 quarterly dividends of $1\frac{3}{4}\%$ sells it at $115\frac{1}{4}$. What was the total amount of his dividend? How much did he gain on the purchase and sale of his stock?

21. Make out a bill for the following:

Bought of the Red Book Co.: 4 gross lead pencils @ $3·25; 6 doz. qt. ink @ $7·20; 40 reams foolscap @ $1·20; 10 gross pen points @ 35c.; 120 lb. blotting-paper @ $7·50 a 100 lb. Discounts 10%, 5%.

22. A note for $300 was dated 1st October, 1909, and drawn for 3 mo. If it was discounted at the bank on 10th November, 1909, at 6%, find the proceeds.

23. A merchant buys a book, the list price of which is $7·50, at a discount of 30% and sells it for $8. What is his gain per cent.?

24. I buy at 120 a stock which pays an annual dividend of 6%. What rate of interest per annum do I get for the money I invest?

25. What must be paid for a sight draft for $1200, if exchange is at $\frac{3}{8}\%$?

26. A grocer buys 45 doz. eggs at 20c. a doz. but finds that 15 eggs are broken. At what price a doz. must he sell the remainder to clear $16\frac{2}{3}\%$ on the total cost?

27. What can one afford to pay for a $6\frac{1}{2}\%$ stock in order that he may receive 5% interest on the money invested?

SQUARE ROOT

When a number is multiplied by itself the product is called the **square** of the number, and the number is called the **square root** of the product. For example, 16 is the *square* of 4, and 4 is the *square root* of 16.

It is evident, that when a number can be factored and its factors arranged in two groups, each containing the same factors, the product of the factors in one of these groups is the square root of the number.

Example: $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$
$= (2 \times 3 \times 3) \times (2 \times 3 \times 3).$

The square root of 324 is $2 \times 3 \times 3$ or 18.

EXERCISE 90

1. What are the squares of: 3, 4, 8, 9, 10, 7, and 6?

2. What are the squares of: 12, 15, 18, 45, and 90?

3. What are the square roots of: 9, 16, 49, 121, 1600, and 8100?

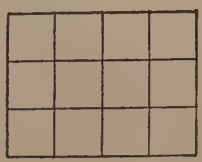
4. What are the square roots of ·25, $\frac{1}{8}\frac{1}{6}$, ·49, 1·44, and $\frac{6}{1}\frac{4}{6}\frac{4}{9}$?

5. What are the lengths of the sides of squares whose areas are: 64 sq. in., ·09 sq. yd., 3600 sq. rods, $\frac{1}{4}$ sq. mile?

6. What is the perimeter of each of the squares whose area is given in Question 5?

7. Find, by factoring, the square root of: 324, 1089, 7921, 9801, 15625, 65536, ·2809, 125·44, 527076.

8. A lot is 81 ft. wide and 169 ft. long. What is the length of the side of a square that has the same area?



9. The width of a lot is $\frac{3}{4}$ of its length. If the lot contains 30 acres, find its perimeter.

The numbers 9, 64, 169, and 625 are *perfect squares*, but the numbers 10, 27, 85, and 113 are not. Why?

Square the numbers 1 and 9, 10 and 99, 100 and 999, 1000 and 9999, and the decimals ·1 and ·9, ·01 and ·99, ·001 and ·999. From the results it will be seen that there are twice as many, or 1 less than twice as many digits in the square as in the number itself. Hence if the digits of a number be separated into periods of two digits each, beginning at the decimal point, the number of periods will be the same as the number of digits in the square root of the number.

Thus: 5/59/32/25 will have 4 integral places, and 41/43/·49/69 will have 2 integral and 2 decimal places.

Knowing this, the value of the highest digit in the square root of any number can be found by inspection.

Example: Find the value of the highest digit in the square root of 4761. Separating the digits into periods we get 47/61, which shows that there will be 2 places in the square root.

Therefore, the highest digit in the square root is a tens' digit. Now 6 tens, or 60, is the greatest ten whose square is contained in 47 hundred. Hence, the tens' digit in the square root is 6.

ORAL EXERCISE

1. How many digits will there be in the squares of integral numbers containing 3, 4, 5, 6, 7 digits?

2. How many digits will there be in the squares of numbers containing 1 integral and 1 decimal place? 2 integral and 1 decimal place? 1 integral and 2 decimal places? 3 integral and 4 decimal places?

3. How many digits will there be in the square roots of numbers containing 3, 4, 5, 7, or 10 integral places?

4. How many digits will there be in the square roots of numbers containing 3, 4, 5, 7, or 8 decimal places?

5. How many digits will there be in the square roots of numbers containing 3 integral and 1 decimal place, 4 integral and 3 decimal places, 5 integral and 5 decimal places?

6. What will be the value and the place of the highest digit in the square roots of: 265, 3708, 4956, 13464, 53872, 6921463, ·2618, 432·15?

7. By looking at the endings of the numbers in Question 6, state which numbers cannot possibly be perfect squares.

In multiplying 37 by itself, the work may be set down as follows:

$$\begin{array}{r} 30+7 \\ 30+7 \\ \hline 900+30\times 7 \\ \quad +30\times 7+49 \\ \hline 900+2\times 30\times 7+49 \end{array}$$

$$=30^2+2\times 30\times 7+7^2=1369.$$

Employing this method, find the squares of 25 and 94.

Similarly $375^2=370^2+2\times 370\times 5+5^2$.

It is evident that if the steps in the above multiplication be retraced, the square root of 1369 will be found. Now the first digit in the square root of 1369 is 3 tens, that is, 30, and so the question is to find the 7.

Squaring the 3 tens and subtracting the result from 1369 there will be a remainder of 469.

From the multiplication it is seen that $469=2\times 30\times 7+49=7\times(60+7)$. So the 7 will be found by dividing 469 by 67. But since the 7 is not yet found an approximate quotient is obtained by dividing by 60, that is, by 2×30. The quotient 7 is then added to the 60 and the sum multiplied by 7 and the result is 469.

Example 1: In finding the square root of 1369 the work may be set down thus:

$$\begin{array}{rl} & 13/69(37 \\ & 9 \\ \hline 67) & 469 \\ & 469 \\ \hline \end{array}$$

Explanation: Separate into periods of two figures each. Find the square root of the greatest square in 13. This is 3, the first figure in the square root required. Subtract this greatest square from 13 and take down the next period and the result is 469. Now take twice the part of the square root already found and the result is 6. Divide 6 into 46 and the quotient is 7. Place the 7 to the right of the 3 and also to the right of the 6. Now multiply 67 by 7 and subtract the product from 469 and there will be no remainder. The required square root is 37.

Example 2: Find to one decimal place the square root of 3246.

```
32/46/·00(56·9
25
106) 746
      636
112·9)110·00
       101·61
         8·39
```

Here the first two digits are found as in Example 1, and these two digits are then considered as a single digit and the work of finding the *third* digit is exactly the same as that done in finding the *second.*

The steps in the methods used to find the square root of a number are as follows:

1. Begin at the decimal point and separate the digits into periods of two figures each.

2. Find the greatest number whose square is contained in the left-hand period, place it at the right as if it were the first digit of a quotient, subtract its square from the left-hand period, and to the right of the remainder add the next period for a dividend.

3. Double the part of the square root already found and place the product at the left for a trial divisor; divide the

dividend, leaving out the right-hand digit, by this trial divisor, and the quotient will be the second digit in the square root.

4. Place the second digit to the right of the trial divisor and multiply the whole divisor thus formed by this second digit; subtract the product from the dividend, and to the right of the remainder place the next period to get the next dividend.

5. Again double the part of the square root now found, use the product for a trial divisor, and find the third digit of the square root as before. Then so proceed until all the periods are taken down.

EXERCISE 91

Find the square root of each of the following numbers:

1. 3844, 4096, 5329, 8464.
2. 19881, 24336, 37249, 65536, 97969.
3. 173056, 277729, 356409, 654481, 772641.
4. 2819041, 7387524, 38576521.
5. ·1296, ·3249, ·6241, ·7744, ·0289, ·0729.
6. 7·29, 11·56, 146·41, 49·1401, 341·1409.
7. ·150544, ·327184, 1218·7081, 5·774409.
8. Find the square roots of the following correct to three places of decimals:

2, 3, 5, 10, 17, 123, ·9, ·51, 6·2, 4·25.

9. Find the square roots of the following fractions:

$\frac{361}{576}$, $\frac{784}{841}$, $\frac{1444}{2401}$, $3\frac{1}{16}$, $25\frac{121}{144}$, $32\frac{164}{625}$

10. Find to three places of decimals the square root of each of the following:

$\frac{3}{4}$, $\frac{5}{7}$, $\frac{3}{11}$, $\frac{4}{27}$, $4\frac{1}{3}$, $3\frac{2}{3}$.

11. If in two years, at interest compounded annually, the amount is 1·1025 of the principal, find the rate per cent. per annum.

12. At what rate per cent. per annum will a sum of money amount to 1·4641 of itself in four years at interest compounded annually?

13. Find, in rods, the perimeter of a square plot of ground containing 8 acres.

14. A rectangular field is 45 rods long and 25 rods wide. Find the length of the side of a square field whose area is the same as that of the rectangular one.

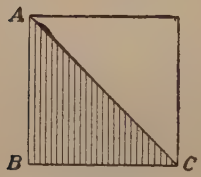
MENSURATION

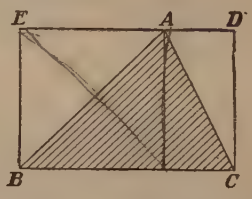
1. AREAS

(1) *Rectangles*

For the measurement of the areas of **rectangles** and **squares** see pages 70 and 71.

(2) *The Triangle*





What part of the whole rectangle is the shaded triangle in each of the above figures?

How is the area of the rectangle found?

The measure of the area of a triangle $= \frac{1}{2}$ *the measure of the base* $\times$ *the measure of the altitude.*

NOTE: The length of the perpendicular from any angular point of a triangle to the opposite side is called the *altitude* of the triangle; and the side upon which the perpendicular is dropped is called the *base*.

EXERCISE 92

1. Find the area of a rectangle whose base is 15 ft. and altitude 8 ft. What will be the area of a triangle having the same base and altitude as the rectangle?

2. Find the area of a triangle whose base is 22 ft. and altitude 11 ft. What will be the area of a rectangle having the same base and altitude as the triangle?

3. Find the area of a triangle whose base is 13 ft. and altitude 7 ft. 4 in.

4. Find the number of feet of inch lumber which will be required to make a triangular floor having a base 18 ft. long and an altitude of 14 ft. 2 in.

5. Look at the walls of your house or barn. What parts of them are triangles? Get the measurements of these triangles. Find how many feet of inch lumber it will require to cover the gable end of the barn, also how much it will cost to paint it at 15c. a sq. yd. How much inch siding will it require for the four walls of your house or barn? How much will it cost to paint the barn?

6. Construct triangles. Measure carefully the lengths of their bases and their altitudes, then find their areas.

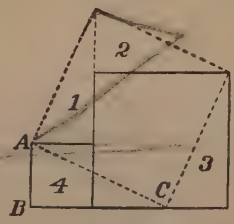
7. The area of a triangle is 12 sq. yd.; its base is 9 ft. Find its altitude.

8. The area of a triangle is 135 sq. in.; its altitude is 1 ft. 6 in. Find the length of its base.

9. The area of a triangle is 60 sq. ft. Find all possible lengths of the dimensions, which can be given in integral feet, of the rectangle whose area equals that of the triangle.

(3) *The Right-Angled Triangle*

In the **right-angled triangle** the side opposite the right angle is called the **hypotenuse.** What names have been given to the other two sides?



On each of the sides of a right-angled triangle, *ABC,* construct a square. Place the two small squares as shown by the heavy lines, and the large square as shown by the dotted lines in the figure. You will see by placing the triangle (1) on the triangle (4) and the triangle (2) on the triangle (3) that:

The square on the hypotenuse = the sum of the squares on the other two sides.

EXERCISE 93

1. The base of a right-angled triangle is 4 ft., the altitude 3 ft. What is the area of the square on the hypotenuse? What is the length of the hypotenuse? Construct the triangle and measure the sides.

2. Find the length of the hypotenuse of a right-angled triangle whose base is 12 ft. and altitude 9 ft.

3. A wall is 15 ft. high and 20 ft. long. Find the distance from the upper right-hand corner to the lower left-hand corner.

4. What will be the length of a ladder required to reach the top of a 16 ft. wall if the foot of the ladder is placed 16 ft. from the wall?

5. Find the length of wire which will reach from the top of a telegraph pole, 20 ft. high, to a point on the ground 24 ft. from the foot of the pole.

6. The hypotenuse of a right-angled triangle is 16 ft. and the base is 6 ft. Find the altitude correct to 2 places of decimals.

7. Mention lines in your school-room which will be the sides of right-angled triangles. Measure two of the lines for any triangle and then calculate the length of the third. Check your answers by measuring the lines.

(4) *The Circle*

What is meant by the diameter of a circle? by the radius? by the circumference?

Mention some objects about you whose surfaces are circles. Measure their diameters and their circumferences. By division find to two decimal places the number of times the circumference contains the diameter.

It will be found that approximately:

The circumference of a circle $= 3\frac{1}{7}$ *times its diameter.*

EXERCISE 94

1. What is the circumference of a circle whose diameter is 14 ft.? 8 ft.? 6 ft.? 20 in.?

2. What is the circumference of a circle whose radius is $6\frac{1}{2}$ ft.? 9 ft.? 12 ft.?

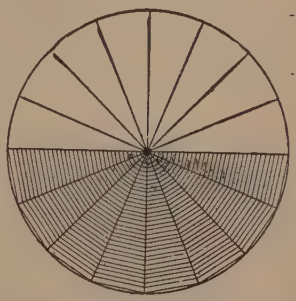
3. What is the diameter of a circle whose circumference is 33 ft.? 18 ft.? 75 ft.?

4. A wheel 4 ft. in diameter will make how many turns in going 2 miles?

5. The diameter of a circular garden is 21 yd. How much farther will a man have to walk to reach the opposite side if instead of crossing the garden he goes around it?

6. A locomotive runs at the rate of 42 miles an hour. The diameter of the driving-wheel is 6 ft. How many turns does the wheel make in a minute? in a second?

(5) *Area of a Circle*



Cut a circle out of leather or stiff paper. Divide it into halves and place one half on top of the other. Now by cutting from the centre of the circle to, *but not through*, the circumference, divide the two halves into the same number of triangular shapes as small as possible. Then open out each half and insert the parts of one half between those of the other, as shown in the figure below.

It will be seen that, if there are parts enough, the figure will become a rectangle.

The measure of the area

$= \frac{1}{2}$ the measure of the circumference of the circle $\times$ the measure of the radius,

$= \frac{1}{2} \times \frac{22}{7} \times 2 \times$ the measure of the radius $\times$ the measure of the radius,

$= \frac{22}{7} \times$ *the square of the measure of the radius.*

EXERCISE 95

1. Find the area of a circle whose radius is 7 ft., 5 ft. 3 in., 19 yds., 24 ft. 6 in.

2. Find the area of a circle whose diameter is 8 ft., 15 ft., 10 ft. 2 in., 12 yd.

3. Find the area of a circle whose circumference is 22 ft., 33 yd., 40 rods.

4. How many acres of land will be inclosed by a circular race-track 1 mile long?

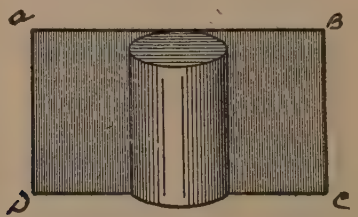
5. Find in yards, correct to two decimal places, the circumference of a circular garden containing $\frac{1}{4}$ of an acre.

6. Find in rods, correct to two places of decimals, the diameter of a circular field containing $5\frac{1}{2}$ acres.

7. Find the area of a circle whose diameter is 14 feet. Give dimensions for rectangles having the same area as the circle.

8. Measure the diameter of a 25-cent piece, of a 50-cent piece. Find the area of each.

(6) *Area of the Cylinder*



How many faces has a cylinder? Name objects which are cylindrical in shape. Is your pencil a cylinder? Is a 5-cent piece a cylinder?

How is the area of the two ends of a cylinder found? If a piece of paper be fitted about the curved face of a cylinder and then unrolled, as shown in the preceding figure, what would be the shape of the paper? How is its area found?

The measure of the area of the curved face of a cylinder is, therefore, found by *multiplying the measure of its circumference by the measure of its altitude.*

EXERCISE 96

1. Find the area of the curved face of a cylinder whose circumference is 10 ft. and altitude 8 ft., circumference $15\frac{1}{2}$ ft. and altitude $5\frac{1}{2}$ ft., circumference 3 ft. and altitude 18 in.

2. Find the area of the curved face of a cylinder when its altitude is 20 ft. and its diameter 4 ft. 8 in.

3. What will it cost at $1\frac{1}{2}$c. a sq. ft. to paint the outside of a cylindrical silo the radius of which is $6\frac{2}{3}$ ft. and whose altitude is 21 ft.?

4. A lawn roller is 3 ft. 6 in. long and 21 in. in diameter. How many square yards of lawn will it cover in making 225 turns?

5. Find the difference, in square inches, between the total area of a cube 7 in. long and the total area of a cylinder 7 in. in diameter and 7 in. in height.

6. The area of the curved face of a cylinder is 396 sq. ft. The altitude is 12 ft. Find the diameter.

7. Find the area of the curved surface of a cylinder the base of which contains $9\frac{5}{8}$ sq. in. and the height of which is 16 in.

2. SOLIDS

We have here the figures of three solids. The first is known as a **rectangular** (in this case **square**) **prism**;

the second is a **triangular prism**; and the third is a **cylinder.**

How is the volume of the rectangular prism found? How can the volume of the triangular prism be found? In both cases the number of cubic units in the volume equals the number of square units in the area of the base multiplied by the number of linear units in the height. The same is true of the cylinder. The general statement is:

The measure of the volume of a prism or cylinder = the measure of the area of the base × the measure of the altitude.

EXERCISE 97

1. What is the volume of a square prism whose altitude is 15 ft. 6 in. and the side of whose base is 4 ft.? Find also the area of its lateral surface.

2. What is the volume of a triangular prism 30 ft. high, one side of whose base is 8 ft. and the length of the perpendicular on it from the opposite angle 5 ft.?

3. What is the volume of a cylinder whose diameter is 12 ft. and whose height is 32 ft.?

4. How many cubic feet of water will a circular cistern hold if the diameter is 6 ft. and the depth $10\frac{1}{2}$ ft.? How many gallons will it hold, there being $6\frac{1}{4}$ gal. in 1 cu. ft.?

5. The water from a flat roof is carried into a rectangular cistern 6 ft. square. If the whole roof is 36 ft. long and 28 ft. wide, how high will a rainfall of $\frac{1}{4}$ in. raise the water in the cistern?

6. If there are 450 cu. ft. in 1 ton of corn fodder, how many tons of fodder can a farmer put into a circular silo 14 ft. in diameter and 20 ft. in height?

7. Take note of triangular prisms, rectangular prisms, and cylinders about you. Make the necessary measurements and find the volumes.

EXERCISE 98

1. A flag pole 48 ft. high casts a shadow 64 ft. long. How far is it, in a straight line, from the top of the pole to the end of the shadow?

2. If there are $1\frac{1}{4}$ bu. in a cubic foot, how many bushels of grain can be stored in a circular tank whose diameter is 30 ft. and whose height is 63 ft.?

3. If a ton of coal measures 40 cu. ft., how many tons can be stored in a rectangular coal shed 60 ft. 4 in. long, 24 ft. wide, and 10 ft. high ?

4. There are $6\frac{1}{4}$ gal. in a cu. ft. In 20 min. how many gallons of water will flow through a $3\frac{1}{2}$ in. pipe if the water is running at the rate of 3 miles an hour ?

5. Find the difference in rods, correct to two decimal places, between the perimeter of a square field containing 10 acres and the circumference of a circular field of the same area.

6. How many square inches of cardboard will it take to make a triangular prism 18 in. long, the end of which is a right-angled triangle whose base is 8 in. and altitude 6 in.? What will be the volume of the prism ?

7. How many cubic inches of lead will it take to make 3 ft. of $2\frac{1}{2}$ in. pipe, the lead being $\frac{1}{4}$ in. in thickness ?

8. Find the area and volume of a cylinder whose diameter is 14 in. and altitude 30 in.

9. A church steeple 72 ft. 6 in. high casts a shadow 58 ft. long. What will be the height of a flag pole which at the same time casts a shadow 64 ft. 8 in. long ?

10. Using a scale $\frac{1}{8}$ of an inch to a foot, draw on a sheet of paper a plan of the whole of the ground-floor of your house. At present prices, find the cost of 1 in. lumber which would be required to make the flooring.

MISCELLANEOUS PROBLEMS

In working problems in Arithmetic there are in every solution two steps with each of which the pupil must make himself familiar.

To illustrate: Find the value of 3720 lb. of wheat at 90c. a bushel.

1st step—Thinking out the solution:

The total number of pounds of wheat is given. The number of pounds in one bushel is known. The number of bushels can, therefore, be found.

The value of one bushel is given; hence the value of any number of bushels can be found.

2nd step—Doing the work:

$$\begin{array}{r} 60)3720 \\ \hline 62 \\ \cdot 90 \\ \hline \$55\cdot 80 \end{array}$$

These are combined thus:

In 60 lb. there is 1 bu.

In 3720 lb. there are $\frac{3720}{60}$ bu.

1 bu. is worth \$0·90

$\frac{3720}{60}$ bu. are worth $\frac{\$0\cdot 90 \times 3720}{60} = \$55\cdot 80.$

EXERCISE 99

1. Read aloud the following: In 1908 the total value of the production of lumber, lath, shingles, cross-ties, poles, and pulpwood in Canada was \$67,425,044.

The production of sawn lumber is shown by the figures to be in the neighbourhood of 3,347,126,000 ft. board measure per annum, valued at \$54,338,036. In this, Ontario leads with a production of 1,294,794,000 ft., valued at \$24,398,077, Quebec being second with 690,135,000 ft., of the value of \$10,838,608, and British Columbia third with 647,977,000 ft.,

worth $9,107,186. The other Provinces rank in the following order: New Brunswick, 308,400,000 ft., valued at $4,081,402; Nova Scotia, 216,825,000 ft., of the value of $2,873,730; Saskatchewan, 91,166,000 ft., valued at $1,576,820; Manitoba, 56,447,000 ft., valued at $867,969; Alberta, 41,382,000 ft., valued at $593,244.

The total production of wood pulp is 363,079 tons, made from 482,777 cords of wood, and valued at $2,931,653.

2. In Question 1 check the accuracy of the amounts 3,347,126,000 ft. and $54,337,036.

3. Taking as correct the figures given in Question 2, find to the nearest cent the average price a thousand at which the lumber was valued.

4. Make out the following bill:

G. S. McKinley, Cobourg, sold C. F. Bayard, Brighton, on 15th April, 1907:

6	doz. gimlets	@ 87½c.
15	doz. garden rakes	@ $3·00
5½	doz. steel shovels	@ $8·50
12	gross screws	@ $1·10
15	doz. 3-in. bolts	@ 75c.
250	lb. nails	@ 5½c.
3½	doz. chisels	@ $8·00
3	doz. hammers	@ $13·50
14	doz. springs	@ $2·50

5. A railway passes through John Miller's farm for a distance of 130 rods, the right of way being 75 ft. wide. At $120 an acre, what is the value of the land taken by the railway?

6. If a boy buys peaches at the rate of 5 for 2c. and sells them at the rate of 4 for 3c., how many must he buy and sell to make a profit of $4·20?

7. A man bought a piece of property for $1,800 and agreed to pay for it in 9 months with interest at 6% per annum. What amount will be due at the expiration of the time?

8. A merchant bought 3 boxes of soap each containing 112 lb. He kept it all summer and it lost $\frac{1}{4}$ of its weight. He then sold it at 13c. a lb. Find his gain if the soap cost him 25c. for 3 lb.

9. Find the cost of papering the walls of a room 20 ft. 9 in. long, 11 ft. 6 in. wide, and 12 ft. 3 in. high, with paper $22\frac{1}{2}$ in. wide, at 15c. a yard.

10. A farmer sells a horse for \$203·50 at a loss of $7\frac{1}{2}\%$. What did the horse cost him?

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11. (*a*) Find the value of 4689 + 6543 + 2847 + 5279 + 7406 + 9975 + 3987 + 6584 + 4968 + 7897.

(*b*) From 6244 subtract 768 and repeat the process till a remainder less than 768 is found.

(*c*) Multiply 798 by 7 and repeat the process until the multiplier has been used 4 times.

12. At \$20 a thousand find the total cost of 55 scantlings 18 ft. long, 4 in. wide, and 3 in. thick; 9 scantlings 14 ft. long, 5 in. wide, and 4 in. thick; 8 scantlings 12 ft. long, 6 in. wide, and 5 in. thick.

13. Pure milk is worth 7c. a quart. If 2 quarts of water be mixed with every 3 gallons of milk, what will a quart of the mixture be worth?

14. Divide \$2400 among A, B, and C so that A will have \$50 more than B and \$70 more than C.

15. Five men agreed to do a piece of work, but two of them did not come, and as a result the work was prolonged $3\frac{1}{2}$ days. In what time could the five men have done the work?

16. At 15c. a yard find the cost of the canvas for a wedge-shaped tent 10 ft. square and 8 ft. high, the sides being 9 ft. 5 in. to the ridge-pole and the canvas being 30 in. wide.

17. A real estate agent bought a lot at 10% below its assessed value and sold it at an advance of 30% above its assessed value, thereby gaining \$810. What per cent. did the dealer make on his investment, and at 20 mills on the dollar what was the amount of the taxes paid on the lot?

18. A hardware merchant bought 29 stoves listed at \$24 each, and received a discount of 8% and 10% off. He sold the stoves at a gain of 20% on the net cost. For what sum did he sell them? If he had sold them at 10% below the list price, what sum would he have gained?

19. A man owing \$9260 is able to pay only 48c. on the dollar. What is his property worth?

20. The average person inhales 30 cubic inches of air at a breath, and breathes 18 times in a minute. When one tenth of a given volume of air is breathed, the whole volume is unfit for breathing. In what time would two persons vitiate the air in a bed-room 15 ft. long, 12 ft. wide, and 10 ft. high?

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21. (*a*) Simplify $(2\frac{2}{9} + 3\frac{11}{12} + 4\frac{3}{8} + 2\frac{5}{6}) \div (3\frac{7}{18} - 2\frac{5}{16})$.

(*b*) Find the value of $39{\cdot}216 \times {\cdot}428 \div {\cdot}0642$.

22. A farmer sells a merchant 6 loads of potatoes weighing with the wagon 3567 lb., 3375 lb., 3742 lb., 3827 lb., 3664 lb., and 3149 lb. If the wagon weighs 1568 lb., and the potatoes are worth 65c. a bushel, how much should the farmer receive?

23. A passenger on a railway train notices that the train passes 6 telegraph poles every 15 seconds. If the poles are 55 yards apart, at what rate in miles an hour is the train running?

24. Frank pledged his watch to a pawnbroker for \$25. At the end of 40 days he redeemed it for \$26·50. What rate of interest did he pay?

25. Four men contracted to do a piece of work for \$8600. The first employed 28 labourers for 20 days of 10 hours each; the second, 25 labourers for 15 days of 12 hours each; the third, 18 labourers for 25 days of 11 hours each; and the fourth, 15 labourers for 24 days of 8 hours each. How much should each contractor receive?

26. Reduce 6 mi. 240 rd. 4 yd. 2 ft. to the decimal of a mile.

27. What will be the value, at \$12 a ton, of the hay in a mow 60 ft. long, 34 ft. wide, and 16 ft. deep, if a cubic foot of the hay weighs $4\frac{1}{2}$ lb.?

28. I bought goods at 25% and 20% off, sold them at an advance of 30% on the net cost, and gained \$475. What sum would I have gained had I sold at 10% advance on the list cost?

29. A merchant insured his stock for $\frac{3}{4}$ of its value at $1\frac{1}{4}$%. The premium was \$91·50. What was the value of the stock?

30. A 60-day note for \$300 without interest is dated 1st June, 1905, and is discounted at the bank on 29th June, 1905, at $5\frac{1}{2}$%. Write the note and find the proceeds.

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31. Divide 375493276 by 8796 and check your answer. What is the fourth remainder in the process? Write it in full.

32. A man sold a pile of 20-inch wood at \$5·20 a full cord. The pile was 80 ft. long and 6 ft. high. He received in payment \$24·25 cash and the balance in flour at \$2·75 per cwt. How much flour did he receive?

33. A tank in the form of a cylinder 8 ft. in diameter is filled with water to the depth of 7 ft. How many gallons does the tank hold? Note: A gallon $= 277\frac{1}{4}$ cu. in.

34. At what fraction of the cost price are goods marked so that the marked price may be lowered 20% and leave a profit of $6\frac{2}{3}$%?

35. A borrows \$450 for which he pays \$2·25 a month. In what time will the interest thereon be 60% of the principal? What is the rate per cent. per annum, simple interest?

36. A dealer has two sorts of tea, one of which he could sell at 75c. a lb. and make 25% on his outlay, and the other at 45c. a lb. and make $12\frac{1}{2}$%. What per cent. profit will he make if he mixes them in equal quantities and sells the mixture at 65c. a pound?

37. A man's income is derived from \$60,000 stock paying a dividend of $3\frac{1}{2}\%$. If he sells his stock at 80 and invests in a 5% stock at 96, how much will his annual income be increased?

38. For 9 bushels of wheat and 7 bushels of corn a farmer received \$11·20. The wheat was worth 32c. a bushel more than the corn. Find the price of a bushel of each kind of grain.

39. The assessed valuation of the real estate of a municipality is \$2,350,640 and that of the personal property is \$876,500. The year's expenses are: For schools, \$10,000; for interest, \$2579·96; for roads, \$8500; for salaries, \$6400; for sinking fund, \$8500; and for other expenses, \$15,000. The municipality receives \$5800 from licenses. What tax must be levied on the dollar to meet the net expenditure?

40. A merchant sends his agent \$8200 instructing him to invest it in city property after deducting a commission of $2\frac{1}{2}\%$. If the property is bought at \$25 a foot frontage, how many feet can the agent buy?

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41. Fill in the following statement of seven weeks' cash receipts and prove the correctness of your work by adding horizontally and vertically:

Week	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Total
1st.....	\$65·91	\$88·74	\$41·82	\$33·60	\$44·18	\$29·41	
2nd	54·36	70·29	36·42	84·72	63·18	98·72	
3rd	81·27	29·82	26·71	21·90	29·16	23·18	
4th.....	54·77	86·41	21·86	90·82	80·27	36·87	
5th.....	81·29	43·81	87·90	21·44	24·33	20·09	
6th.....	29·04	63·83	27·60	86·92	36·81	74·71	
7th.....	40·01	28·42	72·89	27·34	27·42	64·13	
Total							

42. If 70 men can do a piece of work in 6 days working 8 hours a day, in how many days can 15 men do one quarter of the work, working 7 hours a day?

43. A man spent $\frac{1}{3}$ of his money, then $\frac{1}{7}$ more than $\frac{5}{8}$ of the remainder. He now found out that he had still left \$16. How much money had he at first?

44. Find the cost of making a road 4 mi. 120 rd. 11 yd. long at \$760 a mile.

45. A man owes \$3689·20 but can pay only \$2305·75. What can he pay on the dollar?

46. A pile of ordinary brick is 8 ft. 6 in. high, 14 ft. long, and 15 ft. wide. What is the pile worth at \$7·50 a thousand, the dimensions of an ordinary brick being 8 in. $\times$ 4 in. $\times$ 2 in.?

47. A retail dealer sells an article for \$98 gaining 25%. The wholesale merchant sold to the retailer at a profit of 12%, and the manufacturer sold it to the wholesale merchant at a profit of $16\frac{2}{3}$%. What did the article cost the manufacturer?

48. The Mutual Insurance Co. insured a block for \$45,000 at a premium of 75c. a \$100. The Mutual then re-insured \$10,000 of the risk in the Home Co. at a premium of 80c. a \$100, and \$10,000 in the Ætna Co. at a premium of $\frac{5}{8}$%. How much premium did each company receive? What was the Mutual Co.'s net premium?

49. Find the amount of \$3000 in $2\frac{1}{2}$ years at 5% per annum, interest compounded half-yearly.

50. What was the duty on 1250 gallons of spirits invoiced at \$6·40 a gallon, there being a specific duty of \$1·75 a gallon, and an ad valorem duty of 35%? What was the total cost including the duty?

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51. Divide $\frac{7}{9}$ by $\frac{8}{5}$ in two ways giving a reason for each step in the work.

52. Multiply 8765698 by 432728 in such a way that there will be but three partial products. Check your answer.

53. A plot of ground 360 ft. by 132 ft. has a woodshed 60 ft. by 16 ft. built on it, which can be filled to the height of 12 ft. with cord-wood. If 20% of the part of the plot not occupied by the shed is closely piled with cord-wood to the depth of 4 ft., what fraction of the pile will fill the shed?

54. A policeman runs after a thief who has 528 yards start. If the policeman goes at the rate of one mile in 7 min. and the thief at the rate of one mile in 10 min., how far will the thief have gone when he is overtaken?

55. A farmer sells a grain merchant 33,630 lb. wheat at $1·20 a bu., 6,817 lb. oats at 40c. a bu., 16,814 lb. rye at 80c. a bu., 12,600 lb. potatoes at 60c. a bu. Make out the bill, inserting names, place, and date. Receipt it.

56. How much money must be put at simple interest at 8% per annum on 1st Jan., 1908, so that on 15th March, 1911, there will be $198·85 interest due?

57. How many rods of fence will inclose an acre in the form of a square? in the form of a circle?

58. A man sold $\frac{1}{2}$ of a lot of lumber for $\frac{5}{8}$ of what the lot cost. What per cent. was gained on the part sold?

59. Mr. Parker invests $42,892·50 in a 5% stock at 102 paying $\frac{1}{8}$% brokerage. What is his net annual income after paying an income tax of $12\frac{1}{2}$ mills on the dollar on all over $900?

60. Sold two horses for $450 each. On one I gained 20% of its cost and on the other I lost 20% of its cost. Did I gain or lose on both, and how much?

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61. A man bought 24 head of cattle. The average weight of the first three was 975 lb., that of the next four, 1218 lb., of the next nine, 1124 lb., and of the remainder 967 lb. What was the average weight of the whole, and what did the whole cost at $8\frac{1}{2}$c. a lb.?

62. E. Murray bought of Sherwin and Ball, 1st March, 1905: 8 lb. Rio Coffee at $·38; 4 lb. Tea at $·55; $8\frac{1}{2}$ doz. Eggs at $·16; $5\frac{3}{4}$ lb. Butter at $·20; 25 lb. Sugar at $·04$\frac{1}{2}$; 5 lb. Mocha Coffee at $·33; $4\frac{1}{2}$ lb. Italian Prunes at $·12; $\frac{1}{2}$ gal. Maple Syrup at $1·25; $4\frac{3}{4}$ lb. Cheese at $·16; $\frac{3}{4}$ doz. Canned Tomatoes at $1·80. Make out Murray's bill and receipt it.

63. The owner of a house was offered $2500 for it. If the offer had been accepted the loss would have been $16\frac{2}{3}$%. The house was afterwards sold for $3500. Find the gain or loss per cent.

64. What is the rate of taxation on a town whose taxable property is \$850,000, the net tax being \$16,150 after allowing the collector a commission of 5% ?

65. A business firm fails owing W. C. Philips \$3600, J. C. Owens \$4000, and other creditors \$2400. If the firm's net assets were \$7500, how much would Philips and Owens each receive?

66. In 1907 L. Thompson made deposits in his bank as follows: 1st Jan., Balance \$306; 20th Jan., \$480; 3rd Feb., \$95; 26th Feb., \$375; 8th Mar., \$260; 10th Apr., \$508. His withdrawals were as follows: 10th Jan., \$100; 10th Feb., \$210; 17th Mar., \$125; 16th May, \$105.

(*a*) Rule a page for a bank-book and enter the above sums in the proper order and in the proper columns.

(*b*) What was Mr. Thompson's balance on 10th April?

(*c*) If interest at 3% is allowed on all sums from the date of deposit to the date of withdrawal, how would you find the interest on Mr. Thompson's account up to 30th June?

(*d*) Write Mr. Thompson's cheque for any of the sums which he withdrew.

(*e*) Make out a deposit slip for \$95 using bills of different denominations to make up the amount.

67. In calculating interest on ordinary deposits it is the practice in most banks to use the *minimum monthly balance* plan. By this the interest is made up each month, and one calendar month's interest is allowed on the smallest balance on deposit for that month. At the end of the interest period, usually every half year, the interest for the different months is totalled and put to the credit of the depositor's account.

In Question 66 the minimum balance for January was \$206; the January interest was therefore $\frac{1}{12}$ of $\frac{3}{100}$ of \$206 or \$0·51. The February interest was $\frac{1}{12}$ of $\frac{8}{100}$ of \$571 or \$1·43.

Calculate the interest for March, April, May and June, and find the total interest due Mr. Thompson on 30th June.

68. The water that falls upon a flat roof 45 ft. x 66 ft. is carried by pipes into a cistern 10 ft. 6 in. in diameter. What will be the depth of the water in the cistern after a rainfall of $3\frac{1}{2}$ in.?

69. What rate of interest does a man receive on his money who invests it in a $4\frac{1}{2}\%$ stock at 120?

70. A man buys a farm for \$12,500 and pays down $\frac{2}{5}$ of the purchase money. What additional sum ought he to pay at the end of two years, reckoning compound interest at $4\frac{1}{2}\%$ per annum?

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71. (*a*) How often does the square of 181,279 contain $893 \times 441 \div 63$?

(*b*) Simplify: $\dfrac{\cdot5 \text{ of } \cdot4}{\frac{2}{5} \text{ of } \frac{1}{2}} + \dfrac{7\frac{7}{12} \text{ of } 2\frac{2}{5}}{\cdot3 - \cdot2}$.

(*c*) Find the square roots of 15,625 and 146·41.

72. A man owns a lot 297 yd. long and 176 yd. wide around which he wishes to plant a row of elms. The trees are to be placed at equal intervals, one at each corner, and as far apart as possible. How many trees will he require?

73. On Wednesday morning, 27th April, 1910, Harvey Brown commenced to work for George Wilson at \$65 a working month. He boarded with his employer, paying \$3·50 a week. On 27th May, Brown was paid \$35 cash and \$40 on 8th June. What sum was due Brown on 5th July, 1910, if during the term of his employment he lost $4\frac{1}{2}$ days?

74. A man spent $\frac{1}{4}$ of his money for a coat. If the coat had cost \$5 more it would have taken $\frac{1}{3}$ of his money to pay for it. What did the coat cost?

75. What will be the cost of 34 three-inch planks, each 18 ft. long and 10 in. wide at \$18 per M?

76. What must be the face of a note, drawn on 1st June, 1906, for three months without interest, so that when it is discounted at the bank at 5% on 23rd July, 1906, the proceeds may be sufficient to pay a debt of $24·75?

77. What is the face of a bank draft for which I pay $520·65, exchange being $\frac{1}{8}$%? Write the draft inserting names, dates, etc.

78. A school section is assessed for $480,000. The trustees have built a school-house costing $5400. What will the school-house cost a ratepayer whose property is assessed for $6400?

79. A house is worth $14,000 and its contents $10,000. What will it cost to insure both house and contents for 75% of their value, if the premium be $\frac{1}{2}$% and the agent's commission be $\frac{1}{10}$% of the risk?

80. A merchant marked his goods at an advance of 25% on the cost but afterwards sold them at a discount of 8% on the marked price. If his gain was $58·50, find the cost and the marked price.

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81. Find the following results, limiting yourself to 7 minutes, and, when you have finished, check your answers:

(*a*) Add:

82363	32716	58132
14679	14538	42618
93572	76279	73617
46837	68917	83196
2896	24639	64874
734	7249	92786
29	384	46989
9	17	88889

(*b*) Subtract:

97,642	76,213	62,876
23,957	67,476	39,497

(*c*) Multiply:

62·758	14·642
9·27	978

(*d*) Divide: 500,472 by 662.

82. A person bought a certain number of barrels of flour for \$4200. He reserved 40 barrels for his own use and sold $\frac{4}{5}$ of the remainder for \$3260 which was \$60 more than cost. Find the number of barrels bought.

83. A map is drawn on a scale of one inch to 10 miles, and a township is represented on it by a square whose side is $\frac{1}{2}$ in. How many acres are there in the township?

84. A man wishes to have his library arranged on shelves each containing the same number of books. He tries to put 15 books on each shelf and finds that he has 3 left. He then tries 16 books on each shelf and again has 3 left. He next tries 17 books and once more finds 3 remaining. What is the number of books which his library contains? How many books must he put on each shelf, and how many shelves must he have in order to arrange the library as he desires?

85. A can do a piece of work in 5 days, B in 6, and C in 8. In what time can the three together do it?

86. If it costs \$150 to fence a square field at 75c. a rod, what will it cost to plough the field at \$1·25 an acre?

87. It took \$31·20 to redeem a note given 7 years ago with simple interest at 8% per annum. What was the face of the note?

88. How much money must one invest in a 4% stock at 102 in order to have a net income of \$1585 after paying an income tax of 15 mills on the dollar on all over \$600?

89. A Canadian dealer imported 210 yd. of tweed at 6s. 3d. a yd. Find the cost in Canadian money.

90. When \$19 = 80 marks, and 16·1 marks = 20 francs, how many francs are equivalent to \$611·80?

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91. Find the value of:

(*a*) $(4\frac{5}{8} + 1\frac{1}{45} + 5\frac{1}{11\frac{1}{8}}) - (2\frac{1}{8} \div \frac{2}{7}) \times 1\frac{2}{7} + 1\frac{3}{5}$.

(*b*) Find the value of: $\dfrac{(55{\cdot}1 - 41{\cdot}98) \times 4{\cdot}31}{(6{\cdot}842 + 3{\cdot}158) \div {\cdot}125}$.

(*c*) Divide 1839247 by 165 using three factors.

92. For every cent which A gets, B gets $2\frac{1}{2}$c.; and for every dollar B gets, C gets 25c. If amongst them they get $132·00, what is each man's share?

93. A freight car is 36 ft. long, 8 ft. 6 in. wide, and 8 ft. high. How many cubic feet does it contain? If its capacity is 60,000 lb., to what height can it be loaded with wheat? Note: A bushel = 2218·2 cu. in.

94. A contractor undertakes a job that requires the work of 18 men for 15 days to complete it. At the end of 5 days 6 of the men quit work. How many additional men must he employ at the end of the twelfth day in order that the job may be finished within the contract time of 15 days?

95. The end of an iron rod is 3·5 centimetres square, and the rod is 12 metres long. Find its weight to the nearest kilogramme, if a cubic centimetre of iron weighs 7·207 grammes.

96. Carpet of the same grade can be bought $\frac{3}{4}$ yd. wide at 75c. a yard, or 1 yd. wide at $1 a yard. Which width of carpet ought to be purchased to carpet a room, 16 ft. by 14 ft., with least expense? What would this least expense be?

97. If a merchant marks his goods at an advance of 25%, what per cent. discount may he give a customer and still make a profit of 10%?

98. A farmer has 400 bu. beans which he can sell at once at $1·20 a bushel. By storing for 6 months at a cost of $15 paid in advance, he can realize $1·30 a bushel. He takes the latter course. Money being worth 4% per annum, find his gain or loss at the time of sale.

99. Mr. Taylor bought a house for $8000 which rented for $52·50 a month. The taxes were 18 mills on an assessment of $\frac{3}{4}$ of the value of the house, and the annual repairs amounted to $\frac{1}{2}$% of the value of the house. What rate of interest did Mr. Brown make annually on his investment?

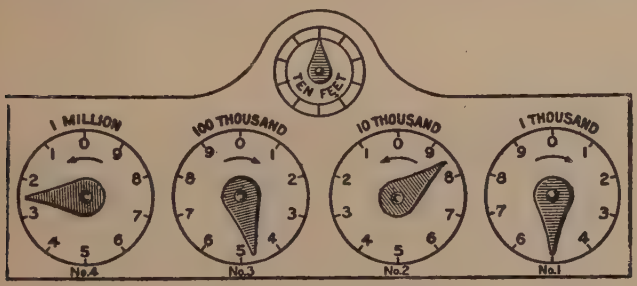
100. T. R. Grover invested $9600 in a 4% stock at $79\frac{3}{4}$. He afterwards sold his stock at $84\frac{1}{4}$ and invested the proceeds in a 6% stock at $104\frac{3}{4}$. If he paid in each case $\frac{1}{4}$% brokerage, find the change in his income.

101. (*a*) Reduce $3\frac{1}{2}$ times £2 4s. to the fraction of £3 4s. 2d.

(*b*) Find, to two places of decimals, the square root of 3762, and of 68253.

102. The following figure shows the four dials of a gas-meter. Dial No. 1 records the number of *hundred* cubic feet of gas; dial No. 2, the number of *thousand* cubic feet; dial No. 3, the number of *ten-thousand* cubic feet, etc.

NOTE: Read the last number which the hand has passed, or that on which it rests.



(*a*) How many cubic feet of gas does the meter register?

(*b*) Find the net cost of the gas at \$1·50 a thousand cubic feet with a discount of $16\frac{2}{3}\%$.

(*c*) Draw the figure to show the position of the hands when 86,900 cu. ft. more gas has passed through the meter.

103. The floor of a room 33 ft. 10 in. long and 26 ft. 6 in. wide is to be tiled with marble squares. Find in inches the length of the largest tile that may be used if there is to be no waste. How many thousand tiles will be required?

104. On counting the apples in a basket by 3 at a time, or by 4 at a time, or by 5 at a time, there are always 2 over; but by counting them 7 at a time, there are none remaining. Find the least number of apples.

105. In 1909, the Champlain Cheese Factory received \$18,591·41 for cheese, and \$63·43 for interest on deposits

in the bank. The factory paid $1828·39 for manufacturing and other expenses. George Crane, one of the shareholders, had, at the end of the year, 71,430 lb. of milk to his credit. If the total amount of milk manufactured was 1,742,323 lb., how much of the factory's profits would Mr. Crane receive?

106. A circular cast-iron band is 14 in. in diameter and $3\frac{1}{2}$ in. thick. The hole in its centre is 7 in. in diameter. Find the weight of the band, if cast-iron is $7\frac{1}{5}$ times as heavy as water.

107. On 10th May, 1907, Wm. Turner gave Henry Sills his note for $425 for four months, with interest at the rate of 5% per annum. Write the note and find its value when due.

108. A Canadian wholesale merchant bought in France 1968·5 metres of silk at 10 francs a metre. He pays 20% ad valorem duty and sells the silk at $2·75 a yard. Find his profit.

109. The amount of a sum for a certain time, at 8% simple interest, is $336, and at $7\frac{1}{2}$% for the same time is $330. Find the sum and the time.

110. On 1st July, 1910, Mr. Merritt deposits $600. On 4th August he withdraws $200; on 15th September deposits $225; on 12th October deposits $350; on 8th November withdraws $175. Using the *minimum monthly balance*, find the total interest at $3\frac{1}{2}$% per annum which the bank will add to Mr. Merritt's credit on 31st December.

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111. Simplify $1\frac{4}{7} - \frac{3}{4}$ of $(\frac{1}{15} + \frac{1}{5}) + \frac{3}{104} \div \frac{2\frac{3}{16}}{3 - \frac{5}{8}}$.

112. At the beginning of the year 1908 the assets of Messrs. Reed, Henry & Co. were: Cash, $2368·25; Merchandise, $8372; Accounts outstanding, $1364·87. Their liabilities were: Notes, $2385; Accounts unpaid, $1694·50. At the end of the year their assets were: Cash, $4576·28; Merchandise, $9465; Accounts outstanding, $2425·90. Their liabilities were: Notes, $3000; Accounts unpaid, $1267·75. Did the firm gain or lose during the year, and how much?

113. The product of four consecutive numbers is 143640. Find the numbers.

114. How many bushels of barley worth 62c. a bushel must a dealer mix with 135 bushels worth 53c. a bushel in order to make a mixture worth 59c. a bushel?

115. The floor of a barn is 40 ft. long, 28 ft. wide, and 3 in. thick. What will the lumber cost at $35 a thousand?

116. On 11th May, 1909, Anderson & Reynolds, of Winnipeg, sold Robert Mason goods as follows: 84 yd. towelling at 18c., 250 yd. table linen at 56½c., 160 yd. cotton at 8¼c., 120 yd. print at 15c., 15½ gross thread at $5. The account is subject to a discount of 25%, and to a further discount of 10% for cash, 5% for payment within 30 days, or 2% for payment within 60 days. Make out the invoice and find the amount that would settle the account on 11th May, 10th June, or 5th July, 1909.

117. Certain liquors are subject to a specific duty of $2·40 a gallon as well as to an ad valorem duty of 30%. If on a shipment of liquor invoiced at $5·40 a gallon the total duty was $253·26, find the number of gallons.

118. For the benefit of his son a man invests $4000 at 5%, the interest to be compounded half-yearly. How much will there be to the son's credit in 3 years?

119. An agent sold 2000 lb. of tea at 60c. a pound. After deducting his commission of 2½% he bought a draft with the proceeds, paying ⅛% exchange. What was the amount of the draft?

120. A 3-month note, dated 10th December, 1909, for $1600, with interest at 4½% was discounted at the bank 3rd Jan., 1910, at 6%. Find (*a*) the day of maturity, (*b*) the value at maturity, (*c*) the term of discount, (*d*) the discount, and (*e*) the proceeds.

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121. (*a*) Simplify $(7\frac{1}{8} + 5\frac{6}{7} - 6\frac{1}{4}) \div (\frac{1}{7} \text{ of } 8\frac{3}{8})$ of $(7\frac{4}{5} \div 8\frac{3}{8})$.

(*b*) Find the G.C.M. of 3127, 3551 and 3975.

122. A boy had to divide 76428 by 123. He copied a figure wrong in the divisor and obtained as his quotient 611 with the remainder 53. What mistake did he make?

123. If for an ocean voyage the cost of a steerage ticket is $\frac{2}{5}$ that of a second-class ticket, and the cost of a second-class ticket is $\frac{2}{3}$ that of the first-class, find the amount saved by travelling steerage instead of second-class on a boat for which a first-class ticket will cost $135.

124. Assuming that a kilogramme is equal to 2·204 lb. and that a franc is equal to 19·3c., find to the nearest cent the value in Canadian money of 10 lb. of tobacco which costs in France 20 francs a kilogramme.

125. A householder uses 16 tons of coal which he can buy on 1st July for $6·25 a ton or on 31st December for $7·00. If money is worth 4% per annum, what will be his gain, on 31st December, if he decides to make his purchase in July?

126. A dealer purchases 85 horses, paying a uniform price for each. He sells 25 of them at a profit of 20%, 40 of them at a profit of 25%, and the remainder at a profit of 10%. If his whole profit is $2975, what did each horse cost him?

127. The price of gold is £3 17s. 10½d. an ounce. How many ounces of pure gold will it require to make 623 sovereigns (pounds) if the value of a sovereign is equal to the value of the gold it contains? What will be the whole weight of the sovereigns if the gold is but $\frac{11}{12}$ of the whole weight?

128. Mr. Rice rents his farm for $480 a year which is 8% of its value. He pays $29 insurance and a tax of 3½ mills on the value of the farm. If he sells his farm and invests the money in an 8% stock at 120, will his annual income be increased or diminished, and by how much?

129. A shipment of grain was insured at $\frac{1}{4}$% to cover 75% of its value. The premium paid was $33·12. If the grain was worth 92c. a bushel, find the number of bushels.

130. A triangular trough is 12 ft. long, 10 in. deep, and 18 in. across the top. How many gallons of water will it contain?

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131. Find the value of:

$$(36 \times 217 \times 528 \times 75) \div (84 \times 396 \times 155 \times 27).$$

132. Find the cost of the poles and wire required for a telephone line 9 miles long, measured from the first to the last pole, if the poles are placed 44 yd. apart and cost \$1·10 each, and if there are two wires each weighing 180 lb. to the mile and costing $4\frac{1}{2}$c. a pound.

133. Two cogged wheels work together. On the one there are 24 cogs and on the other 32. How many revolutions will each wheel make between successive contacts of the same cogs?

134. If 8 lb. of coffee cost as much as 9 lb. of tea, and 5 lb. of tea as much as 48 lb. of sugar, find the cost of 30 lb. of sugar when coffee is worth 45c. a pound.

135. The average age of the pupils in a school of 300 pupils is 9·75 yr. If 20 new pupils are admitted whose average age is 10·35 yr., what is now the total average age?

136. A miller pays 50c. a bushel for barley, 70c. a bushel for pease, and 75c. a bushel for corn. He mixes together 5 bushels of barley, 15 bushels of pease, and 10 bushels of corn, and sells the mixture at \$1·50 a cwt. Find his gain per cent.

137. On 1st Jan., 1907, Mr. Rose borrows from the bank \$1500 at 5% per annum. On 31st Dec., 1907, he pays \$475 for interest and part of the principal. On 31st Dec., 1908, he pays \$625. What must he pay on 31st Dec., 1909, to settle the account?

138. R. H. Fair has \$3200 stock which pays an annual dividend of 3%. He sells out at $86\frac{3}{8}$ and invests in a 4% stock at $114\frac{7}{8}$, the brokerage on each transaction being $\frac{1}{8}$%. Find the alteration in his income and also the total amount of the brokerage.

139. A dry goods merchant sold some cloth at 20% less than the marked price and still made a profit of $8\frac{1}{3}\%$. At what price was the cloth marked if it cost him \$240?

140. Find the depth of a cylindrical cistern containing 4712·4 cu. ft., the diameter of the cistern being 20 ft.

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141. (*a*) Divide 4682973 by 378, using the factors 6, 7, and 9, and show how to get the complete remainder.

(*b*) Find the square root of 537289.

142. A street car makes a trip every hour from 6 a.m. to 12 p.m. and carries on an average 15 passengers a trip. If the fares average 6 for 25c., what will be the car's receipts from 6 a.m. 1st July to 12 p.m. 31st December, omitting Sundays?

143. A can beat B by 5 yd. in a 100 yd. race. B can beat C by 10 yd. in a 200 yd. race. By how much can A beat C in a 400 yd. race?

144. A flour-mill has a capacity of 1200 bbl. a day. How many bushels of wheat will the mill require daily if the weight of the flour is 75% of that of the wheat used? At \$3 a cwt. what is the net daily profit from the flour if the wheat cost \$1 a bushel and the average cost of grinding is 8c. a bushel?

145. If in 146 days \$370 amounts to \$376·66, what is the rate of interest per annum?

146. Rule a page of a bank book. Choosing your own dates and amounts, make eight deposit and six withdrawal entries for any half year. Find the different balances and, using the minimum monthly balance, calculate the interest due on your account at the end of the half year, 30th June or 31st December.

147. Supplying reasonable data for measurements, material, labour, etc., estimate at current prices the cost of: (*a*) carpeting a room; (*b*) painting a house; (*c*) ploughing, seeding, and harvesting a field of grain; (*d*) building a mile of railway: (*e*) building a walk of concrete, brick, or boards; (*f*) drawing earth to level a lawn and sodding the same.

148. A Canadian manufacturer imported a machine which cost in London £85. He paid £3 10s. freight and an ad valorem duty of 30%. What, in Canadian money, was the total cost of the machine?

149. If a cubic decimetre of ice weighs 918 grammes, what is the weight to the nearest kilogramme of a rectangular sheet of ice 53 m long, 27 m wide and 39 mm thick?

150. A barn is 40 ft. long and 32 ft. wide. Its roof projects 1 ft. over the gables and eaves and has a $\frac{3}{8}$ pitch, that is, its height above the level of the eaves is $\frac{3}{8}$ of the width of the barn. How many shingles will it take to cover the roof if 1000 shingles cover 100 sq. ft.?

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151. Define Abstract Number and Applied Number.

152. Why do we begin at the units' place in the addition and the subtraction of numbers? Is it necessary to begin at the units' place in multiplication? Why?

153. What is meant by the prime factors of a number? Distinguish between prime numbers and numbers prime to each other.

154. What different meanings may we give to a fraction?

155. Define: simple fraction, compound fraction, complex fraction, common denominator.

156. How do decimals differ from vulgar fractions? Of what use is the decimal point?

157. Show clearly the correspondence between the addition, subtraction, and division of fractions, and the addition, subtraction, and division of compound denominate numbers.

158. Multiply $\frac{4}{5}$ by $\frac{7}{8}$ and give a reason for each step in the work.

159. A boy on being asked what $\frac{1}{1}\frac{2}{3}$ of a certain fraction was, made the mistake of dividing the fraction by $\frac{1}{1}\frac{2}{3}$ and so got an answer which exceeded the correct answer by $\frac{25}{16^2}$. What was the correct answer?

160. Three numbers of three figures each are multiplied together. What is the greatest and what the least number of figures which may occur in the product?

ANSWERS

EXERCISE

5.— **1.** 22677. **2.** 26723. **3.** 13193. **4.** 5620. **5.** 3539. **6.** 3328. **7.** 4400. **8.** 4412. **9.** 83892.

6.— **1.** \$1908. **2.** \$767. **3.** \$58,160. **4.** 770 ft. **5.** \$8303. **6.** 264,287 lb. **7.** 45,165. **8.** \$40,409. **9.** 189,616 horses. **10.** \$23,296,896. **11.** \$6,210,721. **12.** (*a*) 4,510,070 ac., (*b*) 96,169,700 bu., (*c*) 13,611,237 ac., (*d*) 509,449,824 bu., (*e*) 3,777,960 ac., (*f*) 120,183,379 bu. **13.** 398,184 pupils.

7.— **1.** 1,241,016. **2.** 383,750. **3.** 7,902,936. **4.** 7,119,003. **5.** 934,695. **6.** 2,984,816. **7.** 6,000,667. **8.** 1,125,579. **9.** 338,792.

8.— **1.** 43,078,800. **2.** —. **3.** 87,827 sq. mi. **4.** (*a*) —; (*b*) —. **5.** 3,606,546 sq. mi. **6.** (*a*) —; (*b*) —. **7.** \$805. **8.** \$1330.

9.— **1.** \$3553·66. **2.** \$6217·65. **3.** \$1004·19. **4.** \$1219·78. **5.** \$228·20. **6.** \$70·11. **7.** \$5746·61.

10.—No answers necessary.

11.— **1.** 36, 741, 622, 242, 531, 171, 232, 1551. **2.** 3111, 1003, 2411, 2691. **3.** 112,211. **4.** 623,320; 131,117; 111,510.

12.— **1.** 31. **2.** 185. **3.** 1277. **4.** 977. **5.** 1566. **6.** 992. **7.** 994. **8.** 1322. **9.** 23186. **10.** 23186. **11.** 76814. **12.** 4889. **13.** 14150869. **14.** 5,102,031,064. **15.** 22,016,928,907.

13.— **1.** \$16·27, \$190·75, \$186·25, \$126·70. **2.** \$37,276. **3.** \$2·66. **4.** 139,806 ft. **5.** 84 in. **6.** \$2,289. **7.** 63278. **8.** 32 qt. **9.** North America by 461,947 sq. mi. **10.** 8,735 British cavalry. **11.** B, \$5,339; C, \$13,897; all had \$25,451. **12.** Loss, \$558.

EXERCISE

13.—**13.** 43 yr. **14.** —. **15.** 5 horses. **16.** 2,798,334. **17.** No marbles more. **18.** 379,708. **19.** $2339. **20.** 3. **21.** 236,654,714 lb. **22.** —. **23.** —. **24.** 52. **25.** (*a*) 389, (*b*) 6716, (*c*) 65941, (*d*) 68842. **26.** —. **27.** 55 mi., 228 mi., 323 mi., 448 mi. **28.** Brown lost $23770. **29.** Mon., $10819·28; Tues., $7017·40; Wed., $6455·76; Thurs., $11944·21; Fri., $5917·97; Sat., $6980·67. First wk., $13572·07; second wk., $5697·92; third wk., $11408·04; fourth wk., $8337·95; fifth wk., $10119·31; total, $49135·29. **30.** $2257·36; **31.** —. **32.** —. **33.** $8·42. **34.** —. **35.** $1998·36.

14.— **1.** 13,786; 27,572; 41,358; 55,144. **2.** 141,468; 235,780; 330,092; 424,404. **3.** 114,184; 342,552; 513,828. **4.** 929,274; 1,239,032; 1,548,790; 2,168,306; 2,787,822. **5.** 349,624; 699,248; 786,654. **6.** —. **7.** 12,269,571; 16,359,428; 20,449,285; 24,539,142; 28,628,999; 36,808,713. **8.** 60,070,352; 52,561,558; 90,105,528. **9.** 747,081; 581,063; 498,054. **10.** —. **11.** 937,890; 4,085,370; 392,070.

15.—No answers necessary.

16.—No answers necessary.

17.— **1.** —. **2.** —. **3.** —. **4.** 1,075,927,682; 95,908,764; 28,451,501; 31,673,088; 327,407,970; 722,316,630; 53,224,542; 2,317,467,832; 35,357,740; 18,110,222; 45,771,106. **5.** —. **6.** 544,794,832.

7-15 inclusive—no answers necessary.

18.— **1.** $75, $200, $900, $1225. **2.** $1·84, $4·14, $21·62. **3.** 100,320 ft., 401,280 ft. **4.** 273 da., 1070 da., 4854 da. **5.** 16 oz., 2704 oz., 2316 oz. **6.** 9228 in., 6305 da. **7.** $1008. **8.** $953·60. **9.** $5670. **10.** 288 da. **11.** $2139. **12.** $825. **13.** 156,333 mi.

EXERCISE

18.—**14.** 153,507 rd. **15.** 91,289,796 mi. **16.** 1614 ℔
17. 292,560 ℔. **18.** 5984 ℔. **19.** $685·05.
20. 248 mi. **21.** Merchant owes farmer $121·58.

19.— **1.** —. **2.** $8·75. **3.** 7c. **4.** $19200. **5.** $34·65.
6. 37,756,800. **7.** —. **8.** —. **9.** 82 da., 111 da.
10. (*a*) Wed., 4154; Thu., 3257; Fri., 7780; Sat., 3760. (*b*) 7784, (*c*) 9992, (*d*) 578, (*e*) 597, (*f*) 18951, (*g*) $904·75, (*h*) $742·60, (*i*) $1629·40, (*k*) $831·85, (*l*) $4108·60, (*m*) $1167·60, (*n*) $2498, (*o*) $144·50, (*p*) $298·50, (*q*) $4108·60.
11. $10285. **12.** (*a*) $4275, (*b*) 176,673, (*c*) $5,750,283, (*d*) 385,973,620. **13.** 419,060 ℔.
14. 77,539,764; 45,920,796; 63,318,834; 32,347,335.
15. —. **16.** —. **17.** —.

20.—No answers necessary.

21.—No answers necessary.

22.— **1.** $1250. **2.** 8 pt., 37 gal. **3.** 25 ft., 136 ft.
4. 33 bu. **5.** 29 hr., 90 mi. **6.** 212 da. **7.** 52 wk.
8. 36 horses. **9.** 9 reams, 6 quires. **10.** 66 ft.
11. 527. **12.** 33c., 99c. **13.** 4673. **14.** 117.
15. 23335. **16.** $3\frac{1}{2}$ ft. **17.** 123 da. **18.** 342 toes.
19. 1718. **20.** 65 pieces. **21.** —. **22.** 108 horseshoes, 27 horses.

23.— **1.** 508. **2.** 309. **3.** $564\frac{5}{6}\frac{1}{7}$.
4. $298922\frac{2}{2}\frac{}{7}$. **5.** $6392\frac{1}{6}\frac{7}{0}\frac{7}{4}$. **6.** $18194\frac{3}{3}\frac{7}{9}\frac{4}{0}$.
7. $13618\frac{4}{6}\frac{6}{8}\frac{6}{4}$. **8.** $9011\frac{4}{7}\frac{8}{8}\frac{8}{2}$. **9.** $25715\frac{3}{3}\frac{2}{7}\frac{6}{8}$.
10. $3558\frac{2}{3}\frac{0}{0}\frac{1}{8}$. **11.** $333\frac{2}{8}\frac{2}{3}\frac{2}{0}\frac{2}{7}$. **12.** $31683\frac{1}{2}\frac{9}{4}\frac{2}{7}$.
13. $73908\frac{9}{3}\frac{6}{2}\frac{}{7}$. **14.** $9445\frac{5}{7}\frac{6}{2}$. **15.** $6334\frac{5}{6}\frac{3}{8}$.
16. $9633\frac{2}{5}\frac{7}{9}$. **17.** $4296\frac{4}{8}\frac{8}{8}$. **18.** $100195\frac{4}{6}\frac{6}{0}\frac{3}{1}$.
19. $77077\frac{3}{3}\frac{1}{5}\frac{1}{8}\frac{2}{6}$. **20.** $100073\frac{2}{2}\frac{8}{7}\frac{3}{6}\frac{3}{1}$. **21.** 3746.
22. $49982\frac{8}{1}\frac{6}{9}\frac{9}{7}\frac{}{6}$. **23.** $32926\frac{8}{9}\frac{6}{8}\frac{6}{6}\frac{8}{3}\frac{9}{2}$. **24.** $240870\frac{1}{8}\frac{2}{7}\frac{4}{7}\frac{0}{8}$.
25. $6341\frac{4}{6}\frac{1}{6}\frac{7}{7}\frac{3}{9}\frac{2}{4}$. **26.** $25460\frac{8}{9}\frac{7}{7}\frac{6}{6}\frac{1}{4}\frac{6}{0}$.

EXERCISE

24.— **1.** $301\frac{63}{90}$. **2.** $24953\frac{194}{300}$, $12476\frac{494}{800}$, $1069\frac{3094}{7000}$. **3.** $146\frac{378788}{843000}$, $12\frac{501678}{9870000}$. **4.** $124·90. **5.** $2·73. **6.** —. **7.** 31 suits. **8.** 80 acres. **9.** 7000 times, 1250 times, 1932 times. **10.** 15 mi. **11.** 49. **12.** 14 sheep. **13.** 16 tons. **14.** 25 common years. **15.** $40\frac{1}{2}$ boxes. **16.** 7305 days, allowing for five leap-years; 175,320 hours. **17.** $39940\frac{15}{80}$. **18.** 22 rods. **19.** —. **20.** 40 mi.; 12 hr.

25.— **1-17** inclusive—no answers necessary. **18.** 640 times. **19.** —. **20.** —. **21.** 1619. **22.** $19·50. **23.** Men, $52; women, $234. **24.** $112 for children, $168 for women, $168 for men. **25.** $480, $2080. **26.** 90 bbl. **27.** 442 bu. **28.** 8 payments. **29.** 364 months. **30.** 5280 ft., 12 mi. **31.** $4·50. **32.** $272\frac{60}{75}$.

26.— **1.** (*a*) 225, (*b*) 12. **2.** 3864. **3.** (*a*) $892\frac{1}{2}$, (*b*) 455. **4.** 14c. **5.** 8c. **6.** 14c. **7.** 24. **8.** 112 sq. in.

27.— **1.** A, 9c.; B, 3c. **2.** $1479. **3.** 8 wk. **4.** $29·40; 6 da. **5.** $36; 35 da. **6.** Total, $13693·15. **7.** Total, $876·77. **8.** —. **9.** $489·60. **10.** (*a*) 115 in., (*b*) 4830 in. **11.** —. **12.** 4 times; 2c. **13.** 3 sixths, 2 sixths. **14.** $16. **15.** —. **16.** (*a*) $62·50, (*b*) $300·66, (*c*) $104. **17.** (*a*) Total, $40·24; (*b*) total, $25·96. **18.** 50c. **19.** $450. **20.** $614·40. **21.** $160. **22.** 820347, 752136, 835263. **23.** 384. **24.** $1270·80. **25.** $65. **26.** $155·20. **27.** 234 hr., 373 min. **28.** 12 lb. 11 oz.; 67 hr. 30 min. **29.** (*a*) $10100·16, (*b*) $521955·96, (*c*) 643257, (*d*) 732. **30.** (*a*) 60, 34, 48; (*b*) Total, $187·76. **31.** Quotient, 5; last remainder, 3. **32.** $252 gain, $1092 selling price. **33.** 14789. **34.** 5829120. **35.** 1280 lb. **36.** 30c. **37.** 40 lb. **38.** 1364 cent pieces. **39.** $18000; 30c.; A, $1800; B, $2400; C, $1200. **40.** —. **41.** —. **42.** —. **43.** —.

EXERCISE

28.— **1.** —. **2.** 800,960 oz. **3.** 6 tons 4 cwt. 25 lb. **4.** 33 lb. 1 oz. 1 dr. **5.** 715 stone. **6.** 490,288 dr. **7.** 102,900,000 gr. **8.** 87 lb. 6464 gr. **9.** 10 tons 16 cwt. 61 lb. 2 oz.

29.— **1.** 8 bu. 7 qt. 1 pt. **2.** 145 bu. 2 pk. 1 qt. **3.** 543 qt. **4.** 408 bottles. **5.** \$1·92. **6.** 216 bottles; \$108. **7.** 530 lb. **8.** \$7·56. **9.** 1168 bu. 3 pk. 3 qt. **10.** 235,200 oz. **11.** $1\frac{7}{8}$c., $3\frac{3}{4}$c. **12.** 72 da. **13.** \$14·64. **14.** 2c. an oz.; 8 gal. **15.** \$5·25. **16.** \$12·84 loss.

30.— **1.** 5280 ft., 32660 ft. **2.** 1320 rails, 6600 rails. **3.** —. **4.** 20, 36, 48, 16, 2 and $28\frac{1}{2}$ over.

31.— **1.** 40 rd. 15 ft. **2.** 138 mi. 109 rd. 4 yd. 2 ft. 2 in. **3.** 1710894 in. **4.** 0.

32.— **1.** 114 sq. rd., $7\frac{1}{2}$ sq. yd. **2.** 4840 sq. yd. **3.** 1277 sq. in. **4.** 6 ac. 6 sq. rd. 18 sq. yd. 1 sq. ft. 99 sq. in. **5.** \$462. **6.** \$6560. **7.** 32,897,152 sq. in. **8.** 51200 sq. rd. **9.** 0.

33.— **1.** 874 cu. ft. **2.** 77 cords. **3.** 1 cu. yd. 9 cu. ft. 1576 cu. in. **4.** 5 cords, **5.** 7 cords. **6.** 2 cords. **7.** 5 cu. ft. **8.** 198 cars. **9.** 32 cords. **10.** —.

34.—**1.** 128 da., 110 da., 786 da. **2.** 1 yr. 12 da. 0 hr. 27 min. **3.** (*a*) 4th Oct., (*b*) 23rd Oct., (*c*) 4th Nov. **4.** 2,482,608 min. **5.** 12 da. **6.** 45 mi. **7.** 76 min. **8.** $3\frac{1}{3}$ hr. **9.** 6 da. **10.** 44 mi. **11.** (*a*) \$57, (*b*) \$4, (*c*) \$2·30, (*d*) $28\frac{1}{2}$ da. **12.** \$42·60.

35.— **1.** 9°. **2.** $12\frac{1}{2}$ min. **3.** —. **4.** 22575″. **5.** —.

36.— **1.** \$6·40. **2.** $62\frac{1}{2}$ rm. **3.** 15 bbl. **4.** 24 rm. **5.** (*a*) 36,892,800 times, (*b*) 147,268,800 times.

37.— **1.** 1920d., 1248d., 2943d. **2.** £1 15s. **3.** £38 13s. 9d. **4.** £3 18s. 6d.

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EXERCISE

38.— **1.** 64 yd. **2.** \$48·60. **3.** ½ mi.; 6 hr. 56 min.; 416 min. **4.** 64c. **5.** \$96·25. **6.** \$750. **7.** ¼, ¾, \$562·50. **8.** 30th June. **9.** \$91·80. **10.** 42 cords. **11.** 70c. **12.** A, \$1600; B, \$1800; C, \$2600. **13.** \$1680. **14.** 1001 eggs. **15.** 76. **16.** 62 mi. 97 rd. 3 yd. 1 ft. 8 in. **17.** 84 cu. yd. 16 cu. ft. 82 cu. in. **18.** \$85. **19.** \$3·37½. **20.** 27900 weeds. **21.** 13,097,500 insects. **22.** \$192. **23.** \$37·50. **24.** (1) \$1141803·09; (2) 1318969; (3) 14734037012; (4) $942\frac{2}{9}\frac{3}{2}\frac{3}{7}$. **25.** \$383·21. **26.** —. **27.** —.

39.—**1.** 31 bu. 2 pk. 1 pt. **2.** 143 ton 14 cwt. 57 lb. **3.** 1 yd. 1 ft. 10 in. **4.** 19 mi. 96 rd. 4 yd. 2 ft. 9 in. **5.** £104 19s. 7¼d. **6.** 2 cu. yd. 20 cu. ft. 1206 cu. in. **7.** £78 8s. 6¾d. **8.** 194 mi. 88 rd. 4 yd. 2 ft. 3 in. **9.** 3 ton 3 cwt. 7 lb. 5 oz. **10.** 2 yr. 4 da. 17 hr. 56 min. **11.** 405 deg. 57 min. 51 sec. **12.** 275 sq. yd. 8 sq. ft. 9 sq. in. **13.** 39 yd. 2 ft. 4 in. **14.** 2377 ac. 62 sq. rd. 1 sq. yd. 3 sq. ft. 24 sq. in. **15.** 1057 mi. 346 yd. 2 ft. **16.** 25 cord 5 cord ft. 4 cu. ft. **17.** 34 sq. mi. 112 sq. ac. 5 sq. rd. 6¼ sq. yd. **18.** —. **19.** 545 mi. 1530 yd. **20.** 7174 ton 15 cwt. 58 lb. **21.** 41 ft. 3⅓ in. **22.** 13 mi. 954 yd. 1 ft. 2⅔ in. **23.** $981\frac{3}{3}\frac{1}{8}\frac{}{7}$ lb. **24.** $6\frac{8}{4}\frac{}{3}$ times. **25.** $15\frac{1}{1}\frac{1}{2}\frac{6}{4}\frac{2}{8}\frac{8}{2}$. **26.** 48 parcels and 8 lb. **27.** $51\frac{3}{8}\frac{4}{7}\frac{3}{5}\frac{4}{6}$. **28.** 55 times. **29.** 25 yd. 1 ft. 11 in., 51 yd. 10 in. **30.** $603\frac{3}{7}$. **31.** \$10. **32.** 21,120. **33.** 7 yr. 9 mo. 6 da. 19 hr.

40.—No answers necessary.

41.—No answers necessary.

42.— **1.** 1. **2.** $1\frac{1}{3}$. **3.** 1. **4.** $\frac{1}{4}$. **5.** $\frac{4}{5}$. **6.** $\frac{1}{5}$. **7.** $\frac{2}{3}$. **8.** $1\frac{2}{3}$. **9.** 2. **10.** $1\frac{3}{5}$. **11.** $\frac{5}{6}$. **12.** $1\frac{1}{3}$. **13.** $\frac{3}{4}$. **14.** $\frac{1}{4}$. **15.** $1\frac{1}{16}$. **16.** $\frac{5}{16}$. **17.** $\frac{1}{8}\frac{3}{2}$. **18.** $1\frac{1}{2}$.

EXERCISE

42.—**19.** $\frac{2}{5}$. **20.** $2\frac{1}{8}$. **21.** $1\frac{13}{15}$. **22.** $\frac{13}{20}$. **23.** $\frac{7}{32}$. **24.** $1\frac{5}{8}$. **25.** $2\frac{1}{6}$. **26.** $\frac{5}{6}$. **27.** $\frac{7}{16}$. **28.** $\frac{5}{6}$. **29.** $\frac{1}{30}$. **30.** $1\frac{3}{20}$. **31.** $2\frac{3}{10}$. **32.** $\frac{7}{20}$. **33.** $\frac{13}{14}$. **34.** $1\frac{7}{24}$. **35.** $1\frac{13}{18}$. **36.** $\frac{1}{12}$. **37.** $1\frac{3}{40}$. **38.** $\frac{1}{24}$. **39.** $1\frac{2}{15}$. **40.** $1\frac{1}{18}$. **41.** $\frac{13}{20}$. **42.** $1\frac{7}{40}$. **43.** $1\frac{3}{8}$ bbl. **44.** $\frac{4}{5}$ of farm, $\frac{1}{5}$ of farm. **45.** $1\frac{3}{16}$ mi. **46.** $\$\frac{1}{20}$; 5c. **47.** $\frac{11}{24}$ ac.

43.— **1.** $12\frac{5}{8}$. **2.** $2\frac{1}{5}$. **3.** $5\frac{2}{3}$. **4.** $3\frac{1}{3}$. **5.** $5\frac{1}{6}$. **6.** $2\frac{4}{5}$. **7.** $6\frac{1}{2}$. **8.** $4\frac{1}{8}$. **9.** $4\frac{1}{9}$. **10.** $2\frac{9}{10}$. **11.** $7\frac{11}{12}$. **12.** $4\frac{13}{24}$. **13.** $15\frac{9}{40}$. **14.** $6\frac{1}{70}$. **15.** $25\frac{5}{12}$. **16.** $14\frac{7}{12}$ tons. **17.** $16\frac{7}{8}$ ft. **18.** $8\frac{5}{12}$ ft. **19.** $9\frac{11}{12}$c. **20.** $15\frac{13}{60}$ hr. **21.** $8\frac{3}{4}$ yd. **22.** $105\frac{49}{120}$ bu.

44.— **1.** 1. **2.** 6. **3.** $4\frac{1}{5}$. **4.** $3\frac{1}{3}$. **5.** $5\frac{1}{4}$. **6.** $2\frac{2}{3}$. **7.** 70. **8.** $1\frac{1}{2}$. **9.** 15. **10.** 36. **11.** $4\frac{2}{3}$. **12.** $5\frac{1}{7}$. **13.** 22. **14.** 44. **15.** 33. **16.** 142. **17.** $79\frac{1}{2}$. **18.** 196. **19.** $304\frac{1}{2}$. **20.** $151\frac{1}{2}$. **21.** 380. **22.** \$14·94. **23.** \$90·90. **24.** \$44·80. **25.** $13\frac{3}{4}$ mi. **26.** John, $6\frac{1}{4}$ doz.; Fred., $4\frac{3}{4}$ doz.

45.— **1.** $\frac{1}{6}$. **2.** $\frac{1}{30}$. **3.** $\frac{1}{24}$. **4.** $\frac{2}{7}$. **5.** $\frac{3}{25}$. **6.** $\frac{8}{91}$. **7.** $\frac{13}{35}$. **8.** $\frac{35}{72}$. **9.** $\frac{27}{110}$. **10.** $\frac{2}{45}$. **11.** $\frac{1}{14}$. **12.** $\frac{3}{5}$. **13.** 21. **14.** $19\frac{1}{5}$. **15.** $19\frac{4}{5}$. **16.** $2\frac{1}{4}$. **17.** 6. **18.** $5\frac{15}{52}$. **19.** $\frac{332}{105}$. **20.** $\frac{3}{16}$. **21.** $\frac{27}{65}$. **22.** $1\frac{1}{5}$. **23.** $\frac{27}{40}$. **24.** 20 pt., 7840 min., 54 in., 90 pence. **25.** (*a*) 3 gal. 0 qt. $\frac{4}{5}$ pt, (*b*) 2 yd. 2 ft. $10\frac{1}{2}$ in., (*c*) £7 12s. 1d. **26.** $\frac{1}{3}$ of flock. **27.** $\frac{2}{15}$ of mine. **28.** $1\frac{1}{5}$ ac. **29.** $\frac{1}{6}$ of wages. **30.** By rail, $19\frac{2}{7}$ mi.; by boat, $19\frac{2}{7}$ mi.; by coach, $6\frac{3}{7}$ mi. **31.** $37\frac{1}{2}$ ac. **32.** Form IV, 50; Form III, 50; Form II, 100; Form I, 125; Kindergarten, 25.

46.— **1.** $\frac{33}{50}$. **2.** $\frac{5}{1014}$. **3.** $\frac{4}{9}$. **4.** $\frac{5}{252}$. **5.** $30\frac{1}{8}$. **6.** $10\frac{1}{2}$. **7.** $\frac{17}{45}$. **8.** $\frac{1}{48}$. **9.** $\frac{3}{20}$. **10.** $\$2\frac{221}{440}$. **11.** $\$\frac{7}{20}$. **12.** $2\frac{1}{3}$ bu., 7 bu. **13.** $1\frac{7}{8}$ ac. **14.** $\$\frac{2}{3}$. **15.** $4\frac{1}{2}$ hr. **16.** 46 pieces; $\frac{1}{4}$ in.

EXERCISE

47.— **1.** $\frac{15}{28}$. **2.** $\frac{82}{99}$. **3.** $\frac{27}{80}$. **4.** $\frac{3}{4}$. **5.** $\frac{2}{105}$. **6.** $\frac{3}{8}$.
7. $\frac{10}{27}$. **8.** $\frac{33}{40}$. **9.** $\frac{4}{15}$. **10.** $1\frac{1}{2}$. **11.** 6. **12.** $6\frac{2}{3}$.
13. 14. **14.** 30. **15.** 39. **16.** $\frac{1}{6}$. **17.** $91\frac{13}{18}$. **18.** $31\frac{1}{3}$.
19. 1. **20.** $12\frac{1}{2}$. **21.** $9\frac{3}{8}$. **22.** $\frac{2}{5}$ mi.
23. $3\frac{33}{40}$. **24.** $80\frac{5}{8}$c. **25.** $1·61\frac{1}{4}$.
26. $19\frac{3}{5}$ mi. **27.** 30. **28.** $47\frac{1}{4}$ bu.

48.— **1.** $\frac{3}{8}$. **2.** $1\frac{3}{8}$. **3.** $\frac{2}{3}$. **4.** $\frac{3}{4}$. **5.** $\frac{11}{21}$. **6.** $1\frac{7}{20}$.
7. $\frac{14}{27}$. **8.** $2\frac{17}{32}$. **9.** $8\frac{1}{3}$. **10.** $1\frac{3}{7}$. **11.** $\frac{3}{4}$. **12.** $2\frac{1}{3}$.
13. $1\frac{5}{7}$. **14.** $\frac{11}{15}$. **15.** $\frac{49}{104}$. **16.** 10. **17.** 32. **18.** $10\frac{1}{2}$.
19. 16. **20.** $26\frac{2}{3}$. **21.** 18. **22.** $6\frac{2}{3}$. **23.** 4. **24.** 10.
25. $1\frac{1}{5}$. **26.** $1\frac{1}{56}$. **27.** $\frac{1}{8}$. **28.** 1. **29.** $29\frac{4}{25}$.
30. $17\frac{1}{3}$ yd. **31.** $50. **32.** $3\frac{3}{56}$ hr. **33.** 14 bags.
34. 150 yd. **35.** $\$\frac{2}{3}$, $\$\frac{5}{8}$. **36.** $22\frac{1}{2}$c.
37. 12 rd., $6\frac{9}{11}$ rd., $16\frac{4}{11}$ rd. **38.** 36 da. **39.** $5\frac{37}{41}$ hr.

49.— **1.** $95\frac{1}{3}$ ft. **2.** $58\frac{1}{3}$, $40\frac{5}{6}$. **3.** 48 ft., 18 ft. **4.** 250 ac.
5. $1·98. **6.** $26, $19·50. **7.** 15 cows. **8.** $\frac{2}{3}$; 100 lb.
9. $4·91\frac{3}{4}$. **10.** $41·96\frac{2}{3}$. **11.** $6·25, $25. **12.** $30.
13. Jones $\frac{1}{3}$, Turner $\frac{2}{3}$; Jones $250, Turner $500.
14. 1000 mills; $\frac{1}{50}$; $3750. **15.** $78·90. **16.** 215 yd.
17. $\frac{1}{6}$; $\frac{1}{3}$; $\frac{8}{15}$; $1\frac{7}{8}$ da. **18.** $2\frac{2}{3}$ da. **19.** $9·95. **20.** $4\frac{1}{4}$ hr.
21. 9 and 36. **22.** $13 and $49. **23.** $900. **24.** $\frac{17}{20}$, $\frac{5}{6}$, $\frac{23}{30}$, $\frac{3}{4}$ and $\frac{7}{10}$. **25.** $3\frac{3}{4}$. **26.** $1400. **27.** $21\frac{15}{16}$ bu.
28. 5 ft. **29.** $9000. **30.** 156. **31.** $68000.
32. 240 ac., 180 ac., 150 ac., 330 ac., 900 ac.

50.— **1.** $4\frac{3}{4}$ ac., 9 ac., 12 ac., 20 ac. **2.** —. **3.** $9720.
4. $450. **5.** $683·10. **6.** $1197·60. **7.** 16 rd., 152 rd.
8. 5060 yd. **9.** $89·10. **10.** $9·60. **11.** 300 ft.; $60.
12. 9900 cards; $5·94. **13.** 54 posts. **14.** 88 sq. in.
15. —. **16.** —. **17.** 14400 shingles. **18.** $105.
19. 96 hundreds. **20.** $4·29\frac{1}{3}$. **21.** —. **22.** $38·26\frac{2}{3}$.
23. $20·60. **24.** 3 hr. 18 min. **25.** $108. **26.** $80.
27. 1200 sq. in. **28.** 1 in., 2 in., 3 in., 10 in., n in.

EXERCISE

51.— **1.** 7 strips; 6 yd.; 42 yd.; \$50·40. **2.** \$43·20, \$48. **3.** Lengthwise; \$88. **4.** 42⅛ yd. **5.** 14⅜ times; 9 in. **6.** 31 yd. **7.** 18½ yd.; \$23⅛. **8.** 48 strips; 128 yd.; 16 rolls; \$4·80. **9.** 96 yd. **10.** \$3·20. **11.** \$5·86⅔. **12.** 15 rolls.

52.— **1.** 12½. **2.** 17½. **3.** 16. **4.** 90. **5.** $1\frac{87}{88}$. **6.** $11\frac{33}{88}$. **7.** 11¼. **8.** 9⅛. **9.** 14. **10.** 360 bd. ft. **11.** \$113·40. **12.** \$24. **13.** \$32·50. **14.** 19,200 bd. ft. **15.** 1500 bd. ft. **16.** 6600 bd. ft. **17.** 5 bd. ft.

53.— **1.** 3960 cu. ft. **2.** 220 cu. yd.; \$132. **3.** 168¾ cu. ft. **4.** 126 cu. ft. **5.** 30 tons. **6.** 12⅗ tons. **7.** \$57·60. **8.** 1500 gal. **9.** 360 ft. **10.** 1200 ft. **11.** 240 ft.; 4 ft. **12.** 2 ft. **13.** 211,200 cu. ft.; 1,320,000 gal. **14.** 4536 bricks. **15.** 2800 cu. ft. **16.** 45,375 gal.

54.— **1.** —. **2.** 59391 lb.; $29\frac{131}{000}$ tons; $4242\frac{3}{14}$ lb. **3.** Quotient, 3700; Remainder, 47. **4.** \$16,716,000. **5.** 83 bags. **6.** \$1·02. **7.** \$8·22. **8.** 1920 bu. **9.** 670 bu. **10.** \$1·50. **11.** Total, \$47·11. **12.** 7·51 next morning. **13.** \$1·25. **14.** \$9·78. **15.** 31½ mi. **16.** 66 ft. **17.** Bal. 30th Mar. \$719·71. **18.** \$43·61½. **19.** 400 ft.; \$20. **20.** \$31, \$62, \$93. **21.** \$17·10. **22.** \$1584. **23.** —.

55.—No answers necessary.

56.— **1.** 15. **2.** 17. **3.** 17. **4.** 13. **5.** 12. **6.** 91. **7.** 29. **8.** 512. **9.** 1. **10.** 319. **11.** 31. **12.** 191. **13.** 16. **14.** 13. **15.** 55. **16.** 18. **17.** 66 ft.; 18 times.

57.— **1.** 720. **2.** 1344. **3.** 756. **4.** 1650. **5.** 1050. **6.** 16800. **7.** 720. **8.** 2002. **9.** 144. **10.** 10395. **11.** 360. **12.** 6630. **13.** 8640. **14.** 2340. **15.** 9061. **16.** 4464. **17.** 17442. **18.** 960. **19.** 420 sec. **20.** 146 times. **21.** 106 posts. **22.** 1266.

EXERCISE

58.— **1.** —. **2.** —. **3.** —. **4.** 9. **5.** 9. **6.** 7.
7. 5. **8.** 2. **9.** 13. **10.** 21. **11.** 135. **12.** 42.
13. 180. **14.** 2717. **15.** 1333. **16.** 19. **17.** 17.
18. 33. **19.** 36. **20.** 28. **21.** 17. **22.** 175. **23.** 539.
24. 350. **25.** $\frac{4}{11}$. **26.** $\frac{2}{5}$. **27.** $\frac{10}{17}$. **28.** $\frac{12}{37}$. **29.** $\frac{9}{34}$.
30. $\frac{8}{4}$. **31.** $\frac{13}{14}$. **32.** $\frac{12}{13}$. **33.** $\frac{7}{12}$. **34.** $\frac{85}{65}$. **35.** $\frac{1}{23}$.
36. $\frac{9}{32}$. **37.** $\frac{275}{903}$. **38.** $\frac{64}{101}$. **39.** $\frac{5}{6}$. **40.** $\frac{19}{28}$. **41.** $\frac{80}{77}$.
42. $\frac{8}{4}$. **43.** —.

59.— **1.** —. **2.** —. **3.** —. **4.** $\frac{18}{45}$, $\frac{30}{45}$, $\frac{20}{45}$. **5.** $\frac{75}{105}$, $\frac{42}{105}$, $\frac{55}{105}$.
6. $\frac{385}{462}$, $\frac{84}{462}$, $\frac{198}{462}$. **7.** $\frac{6}{36}$, $\frac{15}{36}$, $\frac{14}{36}$. **8.** $\frac{45}{216}$, $\frac{88}{216}$, $\frac{39}{216}$.
9. $\frac{175}{225}$, $\frac{40}{225}$, $\frac{57}{225}$. **10.** $\frac{225}{720}$, $\frac{280}{720}$, $\frac{324}{720}$, $\frac{390}{720}$.
11. $\frac{450}{2100}$, $\frac{756}{2100}$, $\frac{875}{2100}$, $\frac{490}{2100}$, $\frac{480}{2100}$.
12. $\frac{20}{27}$, $\frac{11}{15}$, $\frac{5}{7}$, $\frac{5}{12}$. **13.** $\frac{18}{65}$, $\frac{5}{28}$, $\frac{7}{39}$, $\frac{8}{52}$. **14.** $2\frac{79}{210}$.
15. $2\frac{3}{8}$. **16.** $1\frac{49}{72}$. **17.** $1\frac{119}{180}$. **18.** $3\frac{41}{72}$. **19.** $2\frac{181}{336}$.
20. $\frac{81}{48}$. **21.** $\frac{228}{325}$. **22.** $\frac{122}{286}$. **23.** $\frac{23}{120}$. **24.** $\frac{41}{192}$.
25. $\frac{158}{605}$. **26.** $\frac{13}{21}$. **27.** $\frac{81}{84}$. **28.** $\frac{401}{1680}$. **29.** 0.
30. $\frac{1}{8}$. **31.** $1\frac{11}{100}$. **32.** $3\frac{11}{90}$. **33.** $17\frac{19}{135}$. **34.** $2\frac{77}{80}$
35. $3\frac{47}{70}$. **36.** $46\frac{71}{360}$. **37.** $34\frac{29}{48}$. **38.** $14\frac{479}{1071}$. **39.** $2\frac{148}{220}$.
40. $14\frac{18}{30}$. **41.** $4\frac{251}{315}$. **42.** $5\frac{15}{66}$.

60.— **1.** —. **2.** —. **3.** —. **4.** —. **5.** $\frac{8}{9}$. **6.** $\frac{5}{28}$.
7. $1\frac{82}{91}$. **8.** $\frac{566}{187}$. **9.** $\frac{11}{108}$. **10.** $\frac{71}{459}$. **11.** \$926·10.
12. \$891. **13.** $2\frac{127}{315}$. **14.** $7\frac{1}{3}$. **15.** $2\frac{1}{8}$.

61.— **1.** —. **2.** —. **3.** $\frac{9}{20}$. **4.** $\frac{82}{45}$. **5.** 4. **6.** 29.
7. 9. **8.** 315. **9.** $\frac{3}{8}$. **10.** $\frac{1}{17}$. **11.** $\frac{1}{2}$. **12.** 26.
13. $44\frac{4}{7}$. **14.** $3\frac{1}{13}$. **15.** $3\frac{3}{32}$. **16.** $1\frac{59}{112}$. **17.** $24\frac{108}{176}$.
18. $3\frac{3}{10}$. **19.** $\frac{4}{21}$. **20.** $7\frac{11}{14}$. **21.** —.

62.— **1.** —. **2.** —. **3.** $\frac{15}{16}$, $\frac{8}{15}$, $\frac{11}{12}$, $\frac{1}{6}$, $2\frac{3}{4}$, $\frac{2}{6}$. **4.** $4\frac{1}{2}$.
5. $3\frac{1}{8}$, **6.** $\frac{5}{8}$. **7.** 20. **8.** $11\frac{1}{4}$. **9.** 35. **10.** $\frac{4}{21}$.
11. 33. **12.** $\frac{1}{9}$. **13.** $54\frac{8}{11}$. **14.** $\frac{3516}{29977}$.
15. $14\frac{2}{15}$. **16.** $\frac{4}{5}$. **17.** $\frac{2}{7}$. **18.** $\frac{1}{8}$. **19.** 2.
20. $2\frac{1}{2}$. **21.** $6\frac{10}{13}$. **22.** $\frac{5}{9}$. **23.** $1\frac{9}{16}$. **24.** $\frac{40}{77}$. **25.** $\frac{5}{9}$.
26. 2. **27.** 1. **28.** $5\frac{5}{8}$. **29.** 10. **30.** $1\frac{1}{138}$. **31.** —.

EXERCISE

63.— **1.** $\frac{16}{34}\frac{9}{77}\frac{4}{7}$. **2.** $\frac{38}{10}\frac{8}{6}$. **3.** $\frac{48}{68}\frac{8}{9}$. **4.** $\frac{4}{5}$. **5.** $5\frac{7}{84}$. **6.** $1\frac{8}{7}\frac{9}{0}$. **7.** $\frac{17}{56}$. **8.** $\frac{21}{23}$. **9.** $\frac{20}{77}$. **10.** $4\frac{1}{8}$. **11.** 45. **12.** $6\frac{7}{8}\frac{8}{6}\frac{1}{4}$.

64.— **1.** $\frac{1}{8}$. **2.** (*a*) $\frac{9}{64}$ bu., (*b*) $11\frac{1}{4}$c. **3.** $\$343\frac{7}{11}$, $\$286\frac{4}{11}$. **4.** $\$264{\cdot}36\frac{3}{2}\frac{5}{0}$. **5.** $\$62\frac{7}{2}\frac{9}{0}\frac{5}{0}$. **6.** $2\frac{2}{5}$ da. **7.** 77 bd. ft. **8.** \$67·80. **9.** (*a*) $\frac{2}{5}$, (*b*) \$12, $\$9\frac{3}{5}$, $\$8\frac{2}{5}$. **10.** $\$8{\cdot}09\frac{1}{5}$. **11.** 108. **12.** $23\frac{1}{2}$ yd. **13.** $3\frac{3}{4}$ ac. **14.** (*a*) 6 mills, 5 mills, 4 mills, $3\frac{1}{2}$ mills, $4\frac{1}{2}$ mills, $4\frac{1}{4}$ mills, $4\frac{7}{8}$ mills, half as many mills; (*b*) \$1·08, \$12·825, $\$13{\cdot}531\frac{1}{4}$. **15.** A, $\$162\frac{2}{5}$; B, $\$142\frac{2}{5}$; C, $\$59\frac{1}{5}$. **16.** $\frac{2}{7}\frac{0}{9}$. **17.** 12 da. **18.** \$20250. **19.** $5\frac{8}{8}\frac{4}{4}\frac{0}{1}$ hr. **20.** $38\frac{1}{2}\frac{3}{0}$ mi. **21.** 600 copies; \$2.

65.— **1.** Three-tenths, seven-tenths, thirteen-tenths, two hundred and forty-six-tenths, three hundred and six *and* eight-tenths, two thousand and one *and* four-tenths.

2. Forty-two-tenths, seventy-five-hundredths, eight hundredths, three hundred and fifty-one-hundredths two hundred and four-hundredths, eighteen *and* sixty-three-hundredths, twenty-seven *and* one-hundredth, thirty *and* eighty-seven-hundredths, sixty *and* three-hundredths, two hundred and eighteen *and* forty-two-hundredths, five hundred *and* five-hundredths, three thousand and one *and* eight-hundredths.

3. Eight hundred and twenty-five-thousandths, four hundred and sixteen-thousandths, three hundred and seven-thousandths, six-thousandths, two *and* four hundred and twelve-thousandths, three *and* five-thousandths, sixty-four *and* one hundred and fifty-seven-thousandths, three hundred and nine *and* forty-three-thousandths, seven hundred *and* nine-thousandths, six thousand and twenty *and* seventy-five-thousandths.

EXERCISE

65.— **4.** Four thousand one hundred and sixty-five ten-thousandths, two hundred and sixteen ten-thousandths, thirty-five ten-thousandths, eight ten-thousandths, seven hundred and fifty-six *and* three thousand one hundred and sixty-eight ten-thousandths, four thousand two hundred and eighteen *and* thirty-two ten-thousandths.

5. ·6, 5·3, 7·2, 600·3, ·84, 7·56, ·09, 304·05, ·007, ·014, 11·857, 6·208, 5209·0027. **6.** 5·0, 3·0, 4·3. **7.** 9·00, 7·00, ·30, ·80, 5·70, 3·64. **8.** 4·000, ·030, 5·000, 3·200, 92·010, 12·000. **9.** ·07, ·015, ·004, 3·2, ·805, ·0327, 16·7, 365·008. **10.** —.

66.— **1.** 3·59. **2.** 711. **3.** 179·47. **4.** 216·692. **5.** 15476·306. **6.** 3283·94. **7.** 118·402. **8.** 140·83. **9.** 788·804. **10.** ·18, 3·88, 24·8. **11.** 8·96, 27·93, 5·887. **12.** 87·64, 15·575, 19·82. **13.** 22·8922, 4966·147. **14.** 45·53783, 997·154, 9·99577, 448·757. **15.** 18·89. **16.** ·849. **17.** 2967·512. **18.** 669·0161. **19.** 1439·0721. **20.** 31·9773. **21.** 83·3165. **22.** 995 mi. **23.** 21·06 thousand cu. ft. **24.** 337·63 ac. **25.** 285·805 mi. **26.** 29·2 parts. **27.** ·098. **28.** ·375 lb.

67.— **1.** 29·6. **2.** 41·31. **3.** 5269·081. **4.** 261·720. **5.** 63·08. **6.** 48·042. **7.** 3228·396. **8.** 2·5935. **9.** ·0174. **10.** 1725·0748. **11.** 257·004. **12.** 25243·75. **13.** 5·02866. **14.** 47·892. **15.** ·01944. **16.** —. **17.** —. **18.** 295·275 in. **19.** 7 oz. **20.** 2772 ft. **21.** 696 lb. **22.** 2·5 pk., 2 pk. 1 gal. **23.** 2 da. 9 hr. 7 min. 12 sec. **24.** 18·8496 ft. **25.** $540. **26.** 39·132 lb. **27.** $550·74. **28.** 53·9 games. **29.** 35. **30.** $47·52.

68.— **1.** 2·5. **2.** 1·72. **3.** ·0033. **4.** ·1222. **5.** 3·3. **6.** 15. **7.** 7·5. **8.** 2·78. **9.** ·5. **10.** 235·68. **11.** 2300. **12.** 3070. **13.** ·3606. **14.** ·0137. **15.** 125. **16.** 2500. **17.** 460000. **18.** ·0016.

EXERCISE

68.—**19.** 200. **20.** ·01014. **21.** ·00377. **22.** ·708. **23.** ·04. **24.** ·0128. **25.** —. **26.** 43·68, ·00375. **27.** ·8, ·05, ·016, ·875, ·0625, ·08, ·45. **28.** 6·7246 +. **29.** ·0365. **30.** ·3529 +. **31.** ·4285 +. **32.** ·0769 +. **33.** 7·3684 +. **34.** 20 lb. **35.** 479 bu. **36.** 32 mi. **37.** 7·4 in. **38.** 36·2 cwt., \$95·93. **39.** \$197·925. **40.** \$·81. **41.** 673·06 + gal. **42.** 28·34 +. **43.** ·679 +. **44.** ·923596+ c. **45.** 10·32 + lb. of milk. **46.** \$720·77+, \$604·558 +.

69.— **1.** $\frac{3}{10}$, $\frac{8}{8}$, $\frac{4}{8}$, $\frac{37}{100}$, $\frac{9}{20}$, $\frac{18}{25}$. **2.** $\frac{1}{20}$, $\frac{1}{40}$, $\frac{1}{1000}$, $\frac{27}{40}$, $\frac{317}{1000}$, $\frac{18}{125}$. **3.** $\frac{59}{80}$, $\frac{7}{125}$, $\frac{7}{400}$, $\frac{337}{6000}$, $\frac{129}{400}$. **4.** $4\frac{3}{4}$, $10\frac{1}{4}$, $12\frac{8}{125}$, $7\frac{11}{40}$, $15\frac{8}{25}$.

70.— **1.** —. **2.** ·1875, ·4375, ·68, ·28125, ·234375, ·92, ·552, ·0112, ·425, ·4875, ·228. **3.** ·6666..., ·8333..., ·6363..., ·6153..., ·4666..., ·8235... ·6111..., ·6956..., ·6428..., ·5446... **4.** ·8574.... **5.** 60c. **6.** ·15625 of stock left, \$1000. **7.** $\frac{7}{8}$; ·875; \$2·10. **8.** ·609375; \$1·95. **9.** ·2; \$·74. **10.** ·625; \$14·50.

71.— **1.** 425 cu. ft. **2.** 373·85 lb. **3.** 6 hr. 41·88 min. or 401·88 min. **4.** 3 in. **5.** ·305... **6.** 16·88... **7.** 96·57.... lb. **8.** Each man \$8, each boy \$1·12. **9.** \$20. **10.** 7·38 lb. **11.** 47·52 tons. **12.** \$3000. **13.** 29·529+ mi. **14.** ·008792. **15.** ·42. **16.** 8·32. **17.** \$137·678. **18.** ·95131c., \$865·06, \$655·77. **19.** 10·87 lb.

72.— **1.** —. **2.** —. **3.** —. **4.** —. **5.** —. **6.** 13·48 in. **7.** \$5·10. **8.** \$1·54. **9.** 12·42 mi., 32·187 Km. **10.** 4584·861 m. **11.** 626·335 m. **12.** 5000 steps.

73.— **1.** ·0000000346 sq. Hm., 3·46 sq. cm. **2.** 46830000 sq. Dm., 46·83 sq. Hm.

EXERCISE

74.— **1.** 6214 cu. dm, 6214000 cu. cm.
2. 3·825 cu. dm, ·003825 cu. m. **3.** 48000 bricks.
4. 192 cu. m. **5.** —.

75.— **1.** —. **2.** —. **3.** 200 dl, ·04 dl. **4.** 9000 l.
5. $3170.

76.— **1.** 1·23 g, 2980000 g, 4·900 g.
2. 25 mg, 32000 mg, 124000000 mg.
3. ·064 Kg, 425000 Kg, 86·3 Kg. **4.** 100 Kg.
5. 257·075 Kg, 565·565 lb. **6.** 63·64841 g.
7. 81·5 francs.

77.— **1.** 306⅔ ft. **2.** ¾ min. **3.** $51·28. **4.** 2·4125 Kg.
5. 19·2 cu. m. **6.** 4500 l. **7.** 544·7 Km. **8.** 150 m.

78.— **1.** 70%, 45%, 80%, 64%, 854%.
2. 16⅔%. 62½%, 66⅔%, 23⅓%, 42⅞%, 425%.
3. —. **4.** 7%, 3¾%, 12½%, ¼%, 350%, 1425%.
5. $\frac{3}{10}$, $\frac{13}{400}$, $\frac{7}{40}$, $\frac{201}{1600}$, $\frac{1}{12}$, ·06, ·0325, ·175, 1·25625, ·08333.... **6.** 45; 63; $23\frac{1}{16}$; $393; 3390 ac.
7. $\frac{27}{80}$, 54%. **8.** 16⅔%, 37½%, 6⅔%, 75%, 22⅔%.
9. 50%, 3⅓%, 83⅓%, 150%, 1080%.
10. 800, 1400, 700, 3000, 4800, 1100. **11.** $360.
12. 2751 marks. **13.** 7500, 9000. **14.** 299.
15. 16¼ lb. **16.** $4500. **17.** $\frac{3}{10}$; 30%. **18.** 4%.
19. 52·59 + %. **20.** 16%. **21.** 98⅓%.
22. ⅛, 16⅔%. **23.** 120%. **24.** 10⅔%.
25. $120, $160, $\frac{23}{20}$, $\frac{4}{3}$. **26.** $22500. **27.** 15%.
28. 25%. **29.** $48. **30.** $400.

79.— **1.** $120. **2.** $60. **3.** $37·50. **4.** $18·75.
5. $1741·50. **6.** $219·45. **7.** $4·32. **8.** 2½%.
9. 1½%. **10.** 1¼%. **11.** $1140, $1100·10.
12. $930, $954·80. **13.** 150 subscriptions.
14. $160. **15.** $31·50, 90c. **16.** $118, $282.
17. $3. **18.** 30,000 yd.; $90; $\frac{81}{80}$. **19.** $60,000.
20. 29,400 lb.; $60.

EXERCISE

80.— **1.** —. **2.** $5\frac{1}{2}$, $8\frac{1}{4}$, $12\frac{1}{2}$, 18. **3.** \$15, \$24·92, \$60, \$176,444·36, \$3170·72. **4.** \$410,000, \$150,000, \$2,750,000, \$9,125,000, \$48,500,000.
5. \$18, \$1782. **6.** \$6. **7.** \$13. **8.** 80%.
9. \$160,875. **10.** \$76,882·81. **11.** \$6606.
12. —. **13.** \$30·58, \$3·62.

14. TAX TABLE FOR RATE OF 4 MILLS.

Assessment	Tax	Assessment	Tax	Assessment	Tax
\$1	\$·004	\$4	\$·016	\$7	\$·028
2	·008	5	·020	8	·032
3	·012	6	·024	9	·036

TAX TABLE FOR RATE OF $3\frac{1}{2}$ MILLS.

Assessment	Tax	Assessment	Tax	Assessment	Tax
\$1	\$·0035	\$4	\$·0140	\$7	\$·0245
2	·0070	5	·0175	8	·0280
3	·0105	6	·0210	9	·0315

TAX TABLE FOR RATE OF $8\frac{1}{4}$ MILLS.

Assessment	Tax	Assessment	Tax	Assessment	Tax
\$1	\$·00825	\$4	\$·03300	\$7	\$·05775
2	·01650	5	·04125	8	·06600
3	·02475	6	·04950	9	·07425

15. \$3·80; \$21·96; \$80·23.

81.— **1.** \$10. **2.** \$3·125. **3.** \$112·50. **4.** \$231.
5. \$12·60. **6.** \$36·70. **7.** \$54, \$234. **8.** \$40.
9. \$160. **10.** —. **11.** \$457·80.
12. (*a*) \$271·62$\frac{1}{2}$, (*b*) \$1717$\frac{2}{64}$. **13.** 25%. **14.** 20%.

82.— **1.** \$105·60. **2.** \$12, \$28, \$3·75, \$12·25, \$22·65.
3. \$57·81, \$486·06.

EXERCISE

82.— **4.** \$90·06 int., \$652.91 amt.;
\$65·24 int., \$4265·24 amt.;
\$118·59 int., \$3843·59 amt;
\$289·04 int., \$6539·04 amt.;
\$242·60 int., \$10242·60 amt.
5. \$300. **6.** \$500, \$2400, \$1200, \$14600.
7. 5 yr. **8.** 6 mo., $3\frac{53}{107}$ yr. **9.** 5%.
10. 4%, 6%, $3\frac{1}{2}$%. **11.** $4\frac{1}{2}$%, 5%, 3%.
12. \$252·77. **13.** 6th Oct., 1910; \$127·51.
14. \$1·83. **15.** \$301·99. **16.** \$507·60.
17. \$138·22. **18.** \$201·10.
19. \$2020, \$1620, \$1636·20, \$1336·20, \$1356·24, \$456·24, \$463·08.

83.— **1.** —. **2.** —. **3.** \$1·25, \$11·25; \$1·22, \$14·03; \$5·15, \$20·60; \$7·50, \$42·50; \$67·50, \$157·50; \$156·50, \$469·50; \$1,593·75, \$2,656·25; \$25·49, \$611·76; \$327·60, \$655·20; \$97·94, \$3,166·86.
4. \$252. **5.** —.
6. \$450, \$1,866·75, \$2,400, \$922·50, \$1,744·20.
7. \$478·80, \$422·14, \$2,650·50, \$2,478·60, \$523·26.
8. 10%. **9.** 35%. **10.** \$20. **11.** \$42.
12. \$2·88. **13.** \$4·50. **14.** \$660·00. **15.** \$2·21.
16. \$7·95. **17.** \$42. **18.** \$31·50. **19.** \$85·50.
20. \$70·19. **21.** \$60·04. **22.** \$202·23.
23. \$128·03.

84.— **1.** \$562·43, 62·43. **2.** \$220·50, \$20·50; \$3787·43, \$787·43; \$2331·83, \$331·83; \$5508·11, \$708·11; \$7086·89, \$1086·89. **3.** \$324·73, \$24·73.
4. \$842·74, \$42·74; \$5657·04, \$657·04; \$2081·21, \$81·21. **5.** (*a*) \$790·08, (*b*) \$5964·58.
6. \$3528·68, \$4620·13, \$4564·66. **7.** \$1226·10.
8. (*a*) \$2·45, (*b*) \$10,377·70. **9.** \$4·41.

EXERCISE

85.— **1.** \$3·75. **2.** \$14·30. **3.** \$180. **4.** \$2560.
5. \$3200. **6.** $\frac{3}{20}\%$. **7.** $2\frac{2}{3}\%$. **8.** \$1905.
9. Loss, \$1125. **10.** \$33·20. **11.** \$4·60.
12. \$4955. **13.** \$2500. **14.** \$4998.
15. $\$3193\frac{17}{31}$, $\$2032\frac{8}{31}$, $\$3774\frac{6}{31}$. **16.** \$10,000. **17.** —.

86.— **1.** 8th April, 1910, \$295·41.
2. 6th Dec., 1909; 63 da., \$3·45, \$396·55.
3. 4th Aug., 1910; 93 da., \$14·01, \$985·99.
4. 11th Oct., 1910; 99 da., \$81·37, \$5,918·63.
5. 5th Oct., 1910; 73 da., \$66, \$5934.
6. 13th Dec., 1910; 146 da., \$36, \$1464.
7. $\frac{441}{36500}$, $\frac{36059}{36500}$, \$3650. **8.** \$1,460. **9.** \$3650.
10. 9th Aug., \$615·21. **11.** \$608·26.
12. 18th Aug., 1910; \$2531·85, 76 da., \$36·90, \$2494·95. **13.** 17th Dec., 1908; \$1220·55, 125 da., \$20·90, \$1199·65. **14.** \$3650, 63 da., \$42, $\$243\frac{1}{3}$, $\frac{1}{15}$, $6\frac{2}{3}\%$. **15.** $6\frac{1}{4}\%$.

87.— **1.** \$2500, \$6025; \$6000, \$4455; \$5700, \$7210·50; \$2300, \$621. **2.** 30, 41, 55, 29.
3. \$216, \$1150, \$308, \$850.
4. \$500, \$1500, \$2500, \$30,000.
5. \$5, \$4·50, \$7, \$8·25.
6. (*a*) $4\frac{1}{2}\%$, (*b*) 7%, (*c*) $4\frac{1}{4}\%$, (*d*) $5\frac{1}{3}\%$.
7. 26. **8.** \$350. **9.** \$3400, \$6·25. **10.** \$1233.
11. \$640. **12.** \$480. **13.** \$100·80.
14. 180 shares, \$18000, \$760.
15. 40 shares, \$240, $\frac{1}{20}$, 5%. **16.** $6\frac{2}{3}\%$. **17.** 8%.
18. \$9 larger. **19.** \$4600. **20.** \$9000, \$7200.
21. \$160. **22.** \$150. **23.** \$30. **24.** 40.
25. 75. **26.** —. **27.** $7\frac{1}{2}\%$, \$375, \$525.
28. 10%, \$600, \$750, \$900. **29.** A \$8000, B \$7000, C \$5000; A \$320, B \$280, C \$200.

EXERCISE

88.— **1.** \$4·13, \$14·10, \$26·60, \$30·23, \$47·90, \$75·24, \$93·60. **2.** A 40c. note and 5c. stamp, total cost 46c.; a 90c. note, total cost 97c.; a \$1·50 note, total cost \$1·67; a \$2·50 note and 40c. note, total cost \$2·96; a \$3 note and 70c. note, total cost \$3·84; a \$4 note and 80c. note, total cost \$4·90; a \$5 note and 90c. note, total cost \$5·97; a \$5 note and \$4 note, total cost \$9·21; two \$10 notes, a \$5 note and a \$1 note, total cost \$26·11.
3. 2c. **4.** \$360·45, \$2002·50, \$236·80.
5. \$1201·50. **6.** \$2406. **7.** \$159·80. **8.** \$3208.
9. \$720. **10.** \$320. **11.** \$140. **12.** $\frac{1}{8}\%$.
13. \$6034·46, \$675·50, \$1144·80.
14. £1315·10 +, 23471·40 + francs, 33542·97 + marks.

89.— **1.** Gain \$50. **2.** \$365·40, \$157·50, \$812, \$265.
3. $77\frac{1}{7}\%$. **4.** \$680.
5. $33\frac{1}{3}\%$, $44\frac{4}{9}\%$, $22\frac{2}{9}\%$, \$800, $\$1066\frac{2}{3}$, $\$533\frac{1}{3}$.
6. \$2926. **7.** \$1500. **8.** $29\frac{3}{5}\%$.
9. \$1180, \$1150·50. **10.** \$5040.
11. \$115·41, \$126. **12.** \$24. **13.** \$320·50.
14. \$16·10. **15.** \$590·83. **16.** \$2·40.
17. \$278·78. **18.** 8800 bu. **19.** \$55·25.
20. \$700, \$950. **21.** \$99·78. **22.** \$297·29.
23. $52\frac{8}{21}\%$. **24.** 5%. **25.** \$1204·50. **26.** 24c.
27. \$130.

90.— **1.** —. **2.** 144, 225, 324, 2025, 8100. **3.** 3, 4, 7, 11, 40, 90. **4.** ·5, $\frac{1}{6}$, ·7, 1·2, $\frac{8}{13}$. **5.** 8 in., ·3 yd., 60 rd., $\frac{1}{2}$ mi. **6.** 32 in., 1·2 yd., 240 rd., 2 mi. **7.** 18, 33, 89, 99, 125, 256, ·53, 11·2, 726. **8.** 117 ft., **9.** 280 rd.

91.— **1.** 62, 64, 73, 92. **2.** 141, 156, 193, 256, 313.
3. 416, 527, 597, 809, 879. **4.** 1679, 2718, 6211.

EXERCISE

91.— **5.** ·36, ·57, ·79, ·88, ·17, ·27. **6.** 2·7, 3·4, 12·1, 7·01, 18·47. **7.** ·388, ·572, 34·91, 2·403. **8.** 1·414, 1·732, 2·236, 3·162, 4·123, 11·090, ·948, ·714, 2·489, 2·061. **9.** $\frac{19}{24}$, $\frac{28}{29}$, $\frac{38}{49}$, $1\frac{3}{4}$, $5\frac{1}{12}$, $5\frac{17}{26}$. **10.** ·866, ·845, ·522, ·384, 2·121, 1·914. **11.** 5%. **12.** 10%. **13.** 143·108 rd. **14.** 33·541 rd.

92.— **1.** 120 sq. ft., 60 sq. ft. **2.** 121 sq. ft., 242 sq. ft. **3.** $47\frac{2}{3}$ sq. ft. **4.** $127\frac{1}{2}$ bd. ft. **5.** —. **6.** —. **7.** 24 ft. **8.** 15 in. **9.** 1 ft. by 60 ft., 2 ft. by 30 ft., 3 ft. by 20 ft., 4 ft. by 15 ft., 5 ft. by 12 ft., 6 ft. by 10 ft.

93.— **1.** 25 sq. ft., 5 ft. **2.** 15 ft. **3.** 25 ft. **4.** 22·62.... ft. **5.** 31·24.... ft. **6.** 14·83.... ft. **7.** —.

94.— **1.** 44 ft., $25\frac{1}{7}$ ft., $18\frac{6}{7}$ ft., $62\frac{6}{7}$ in. **2.** $40\frac{6}{7}$ ft., $56\frac{4}{7}$ ft., $75\frac{3}{7}$ ft. **3.** $10\frac{1}{2}$ ft., $5\frac{8}{11}$ ft., $23\frac{19}{22}$ ft. **4.** 840 turns. **5.** 12 yd. **6.** 196 turns, $3\frac{4}{15}$ turns.

95.— **1.** 154 sq. ft., $86\frac{5}{8}$ sq. ft., $1134\frac{4}{7}$ sq. yd., $1886\frac{1}{2}$ sq. ft. **2.** $50\frac{2}{7}$ sq. ft., $176\frac{11}{14}$ sq. ft., $11{,}694\frac{4}{7}$ sq. in., $113\frac{1}{7}$ sq. yd. **3.** $38\frac{1}{2}$ sq. ft., $86\frac{5}{8}$ sq. yd., $127\frac{3}{11}$ sq. rd. **4.** $50\frac{10}{11}$ ac. **5.** 123·33.... yd. **6.** 33·46.... rd. **7.** 154 sq. ft., 1 ft. by 154 ft., 2 ft. by 77 ft., 7 ft. by 22 ft., 14 ft. by 11 ft. **8.** —.

96.— **1.** 80 sq. ft., $85\frac{1}{4}$ sq. ft., $4\frac{1}{2}$ sq. ft. **2.** $293\frac{1}{3}$ sq. ft. **3.** $13·20. **4.** $481\frac{1}{4}$ sq. yd. **5.** 63 sq. in. **6.** $10\frac{1}{2}$ ft. **7.** 176 sq. in.

97.— **1.** 248 cu. ft., 248 sq. ft. **2.** 600 cu. ft. **3.** $3620\frac{4}{7}$ cu. ft. **4.** 297 cu. ft., $1856\frac{1}{4}$ gal. **5.** 7 in. **6.** $6\frac{38}{45}$ tons. **7.** —.

EXERCISE

98.— **1.** 80 ft. **2.** 55687½ bu. **3.** 362 tons. **4.** 2205⅝ gal. **5.** 18·176 rd. **6.** 480 sq. in., 432 cu. in. **7.** 63 9/14 cu. in. **8.** 1628 sq. in., 4620 cu. in. **9.** 80 ft. 10 in. **10.** —

99.— **1.** —. **2.** —. **3.** $16·23. **4.** $238·70. **5.** $443·18 2/11. **6.** 100 doz. **7.** $1881. **8.** $4·76. **9.** $21·07. **10.** $220. **11.** (*a*) 60,175, (*b*) 8 times and remainder 100, (*c*) 1,915,998. **12.** $28·80. **13.** 6c. **14.** A, $840; B, $790; C, $770. **15.** 5¼ da. **16.** $5·37. **17.** 44⅘%, $40·50. **18.** $691·5456, $50·112. **19.** $4444·80. **20.** 4 hr. 48 min. **21.** (*a*) 12⅔, (*b*) 261·44. **22.** $129·09. **23.** 45 mi. **24.** 54¾%. **25.** $2686+, $2158·39+, $2374·23+, $1381·37+. **26.** 6·752651+. **27.** $881·28. **28.** $1319⅘. **29.** $9760. **30.** $298·42. **31.** 7999 less. **32.** 3 cwt. **33.** 2193 9/11 8/10 7/9 gal. **34.** ⅘. **35.** 10 yr., 6%. **36.** 30%. **37.** $400. **38.** Wheat 84c., corn 52c. **39.** 14 mills. **40.** 320 ft. **41.** $2152·37. **42.** 8 da. **43.** $149⅓. **44.** $3329·75. **45.** 62½c. **46.** $361·46¼. **47.** $60. **48.** Mutual, $337·50; Home, $80; Ætna, $62·50; net premium, $195. **49.** $3394·22. **50.** $4987·50, $12987·50. **51.** —. **52.** Partial products, 70125584, 631130256, 3786781536. **53.** 8/9 0/7. **54.** 7/10 mi. **55.** $1119. **56.** $776·76. **57.** 50·596 rd., 44·848 rd. **58.** 25%. **59.** $2085. **60.** $37·50 loss. **61.** 1068 17/24 lb.; $2180·16½. **62.** $13·80. **63.** 16⅔% gain. **64.** 20 mills. **65.** $2700, $3000. **66.** (*b*) $1589. **67.** $14·42. **68.** 10 ft. **69.** 3¾%. **70.** $8190·1875. **71.** (*a*) 5,257,091, (*b*) 15, (*c*) 125, 12·1. **72.** 86 trees. **73.** $28·75. **74.** $15. **75.** $27·54. **76.** $24·90. **77.** $520. **78.** $72. **79.** $162. **80.** $390, $487·50.

EXERCISE

99.—**81.** —. **82.** 840 bbl. **83.** 16,000 ac. **84.** 4083 books, 1361 books, 3 shelves. **85.** $2\frac{2}{9}$ da. **86.** \$19·53$\frac{1}{2}$. **87.** \$20. **88.** \$40,800. **89.** \$319·37$\frac{1}{2}$. **90.** 3200 francs. **91.** (*a*) $2\frac{71}{100}$, (*b*) ·70684, (*c*) $11146\frac{157}{1000}$. **92.** A, \$32; B, \$80; C, \$20. **93.** 2448 cu. ft., 4·195 ft. **94.** 20 men. **95.** 105·5229 Kg. **96.** 1 yd. width, \$26·66$\frac{2}{3}$. **97.** 12%. **98.** \$15·10 gain. **99.** $6\frac{1}{10}$%. **100.** \$96. **101.** (*a*) $\frac{2}{5}$, (*b*) 61·33, 261·25. **102.** (*a*) —, (*b*) \$310·62$\frac{1}{2}$, (*c*) —. **103.** 2 in., 32·277 thousand. **104.** 182. **105.** \$689·83. **106.** $105\frac{35}{128}$ lb. **107.** 432·34. **108.** \$1361·08. **109.** \$240, 5 yr. **110.** \$10·66. **111.** $1\frac{3}{8}$. **112.** \$4173·81 gain. **113.** 18, 19, 20, 21. **114.** 270 bu. **115.** \$117·60. **116.** \$178·92, \$188·86, \$194·82. **117.** 63 gal. **118.** \$4638·77. **119.** \$1168·54. **120.** (*a*) 13th Mar., 1910; (*b*) \$1618·35; (*c*) 69 da.; (*d*) \$18·35; (*e*) \$1600. **121.** (*a*) $6\frac{15}{128}$; (*b*) 53. **122.** Copied 5 instead of 3 in divisor. **123.** \$54. **124.** \$17·51. **125.** \$10. **126.** \$175. **127.** 160 oz.; $174\frac{4}{11}$ oz. **128.** Diminished by \$30. **129.** 6000 bu. **130.** $93\frac{3}{4}$ gal. **131.** $2\frac{3}{5}$. **132.** \$542·90. **133.** 4 and 3 revolutions respectively. **134.** \$1·25. **135.** 9·7875 yr. **136.** $24\frac{6}{11}$ %. **137.** \$556·50. **138.** No change; \$7. **139.** \$325. **140.** 14·99 + ft. **141.** (*a*) 12388; remainder, 309; (*b*) 733. **142.** \$1777·50. **143.** 39 yd. **144.** 5226 $\frac{3}{8}$ bu.; \$1412·20. **145.** $4\frac{1}{2}$%. **146.** —. **147.** —. **148.** \$554·80. **149.** 51233 Kg. **150.** 17,640 shingles. **151** to **158** inclusive—no answers necessary. **159.** $\frac{8}{9}$. **160.** 9 figures, 7 figures.